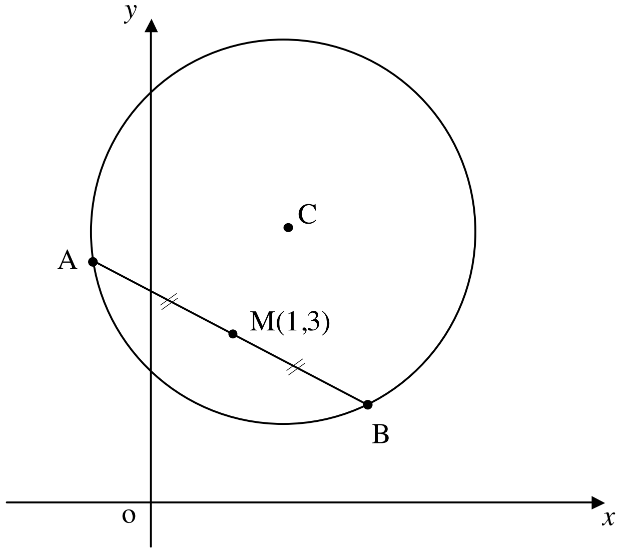
Higher Circles Exam Revision

1. The circle below, centre C, has as its equation $x^2 + y^2 - 4x - 10y + 19 = 0$.
M(1,3) is the mid-point of the chord AB.



(a) Write down the coordinates of C, the centre of the circle. **1**

(b) Show that the equation of the chord AB can be written as $x = 7 - 2y$. **3**

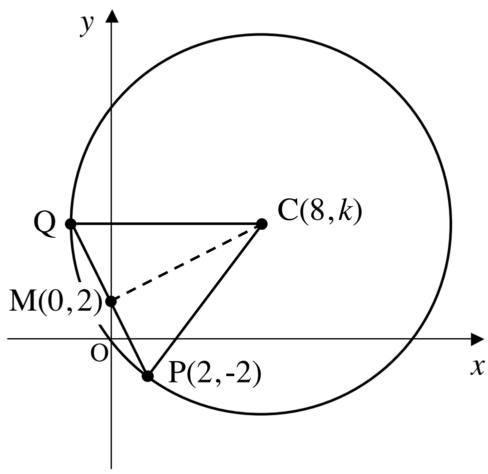
2. A circle, centre C(8,k), has the points P(2,-2) and Q on its circumference as shown.

M(0,2) is the mid-point of the chord PQ.

(a) Find the coordinates of Q. **1**

(b) Given that radius CQ is horizontal, write down the value of k, the y-coordinate of C. **1**

(c) Hence establish the equation of the circle. **3**

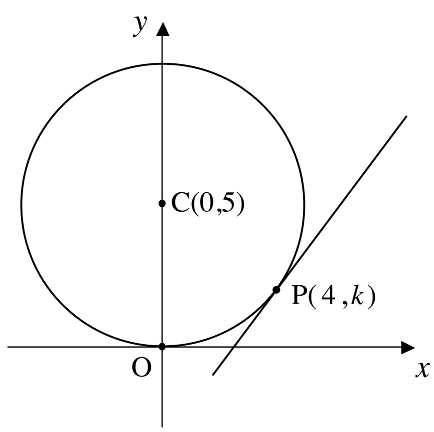


3. A circle passes through the origin and has the point C(0,5) as its centre.

(a) Establish the equation of this circle giving your answer in **expanded form**. **4**

(b) The point P(4,k) lies on the circumference of this circle as shown.
Find **algebraically** the value of k. **5**

(c) Find the equation of the tangent to the circle at P. **3**



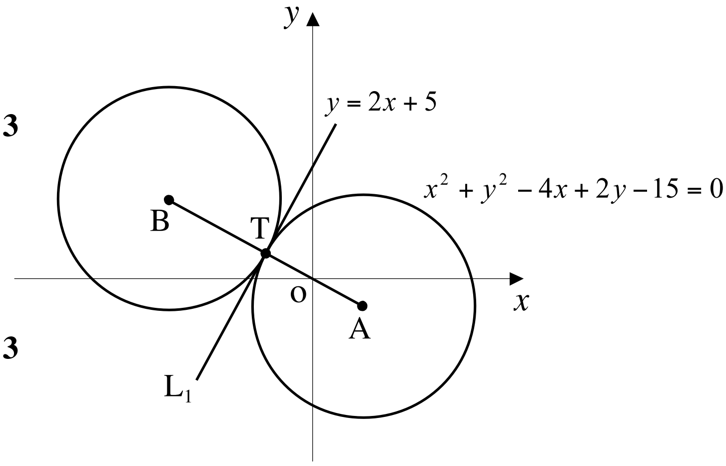
4. Two circles, **both with the same radius**, touch extenally at T as shown below.

The circle with A as its centre has equation $x^2 + y^2 - 4x + 2y - 15 = 0$.

Line L_1 is the common tangent to both circles through T and has as its equation $y = 2x + 5$.

(a) Find the coordinates of T, the point of tangency. **3**

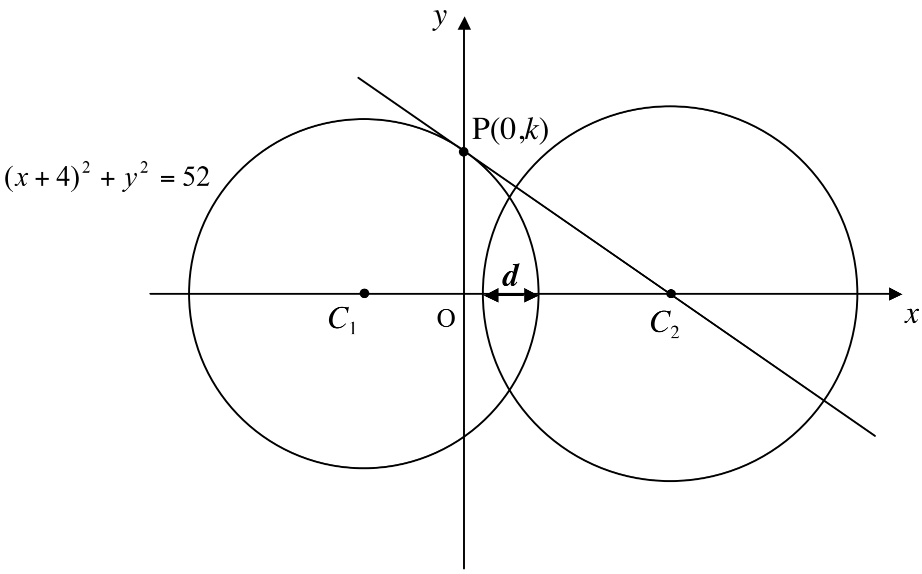
(b) Find the coordinates of B and hence write down the equation of the other circle in the diagram. **3**



5. Consider the diagram below.

The circle centre C_1 has as its equation $(x + 4)^2 + y^2 = 52$.
The point P(0, k) lies on the circumference of this circle and the tangent to this circle through P has been drawn.

A second circle with centre C_2 is also shown.



(a) What is the value of k? **2**

(b) Hence find the equation of the tangent through P. **4**

(c) The tangent through P passes through C_2 the centre of the second circle. State the coordinates of C_2. **1**

(d) Given that the second circle has a radius of 8 units, calculate the distance marked ***d*** on the diagram, giving your answer correct to 1 decimal place. **3**

Question 1

(a) •1 C(2,5)

(b) •1 $M_{cm} = \frac{5-3}{2-1} = 2$

•2 $M_{AB} = -\frac{1}{2}$

•3 $y-3 = -\frac{1}{2}(x-1)$

$2y - 6 = -x + 1$

$x = 7 - 2y$

(c) •1 $(7-2y)^2 + y^2 - 4(7-2y) - 10y + 19 = 0$

•2 $5y^2 - 30y + 40 = 0$

•3 $(5(y-4)(y-2) - 0 \therefore y = 4 , y = 2$

•4 $y = 4$ *then* $x = -1$, $y = 2$ *when* $x = 3$

Question 2

(a) •1 Q(-2,6)

(b) •1 k = 6

(c) •1 strategy

•2 r can be found from horiz. line but some pupils will use points P and C.

$r^2 = 6^2 + 8^2 = 100$

•3 $(x-8)^2 + (y-6)^2 = 100$

Question 3

(a) •1 $r = 5$

•2 $(x-a)^2 + (y-b)^2 = r^2$

•3 $(x-0)^2 + (y-5)^2 = 25$

•4 $x^2 + y^2 - 10y + 25 - 25 = 0$ ....

(b) •1 $4^2 + k^2 - 10k = 0$

•2 $k^2 - 10k + 16 = 0$

•3 $(k-8)(k-2) = 0$

•4 $\therefore k = 8$

•5 $k = 2$

(c) •1 $m_r = \frac{2-5}{4-0} = -\frac{3}{4}$

•2 $m_{\tan} = \frac{4}{3}$

•3 $y - 2 = \frac{4}{3}(x-4)$

Question 4

(a) ans: (-2,1) (3 marks)

$\_^{1}$ substitutes eq.of line in eq. of circle	$\_^{1}$ $x^2 + (2x+5)^2 - 4x + 2(2x+5) - 15 = 0$
$\_^{2}$ simplifies and solves for x	^{2} $5(x+2)^2 = 0; x = -2$
$\_^{3}$ substitutes to find y	$\_^{3}$ $y = 2(-2) + 5$; $y = 1$

(b) ans: $(x+6)^2 + (y-3)^2 = 20$ (3 marks)

$\_^{1}$ establishes coordinates of B	$\_^{1}$ B(-6,3)
$\_^{2}$ finds r^2	$\_^{2}$ $r^2 = 20$
$\_^{3}$ substitutes into general circle equation _	^{3} $(x+6)^2 + (y-3)^2 = 20$

Question 5

(a) ans: $k = 6$ (2 marks)

$\_^{1}$ knows to substitute point	$\_^{1}$ $(0+4)^2 + k^2 = 52$
$\_^{2}$ establishes value of k	$\_^{2}$ $k = 6$

(b) ans: $y = -\frac{2}{3}x + 6$ (4 marks)

$\_^{1}$ finds coordinates of C_1	$\_^{1}$ $C(-4, 0)$
$\_^{2}$ finds gradient of radius	$\_^{2}$ $m_{C_1P} = \frac{6}{4} = \frac{3}{2}$
$\_^{3}$ finds gradient of tangent	$\_^{3}$ $m_{\tan} - \frac{2}{3}$
$\_^{4}$ substitutes into formula	$\_^{4}$ $y = -\frac{2}{3}x + 6$

(c) ans: $C_2(9, 0)$ (1 mark)

$\_^{1}$ subs point, solves for x and states point	$\_^{1}$ $0 = -\frac{2}{3}x + 6$; $x = 9$; $(9, 0)$

(d) ans: 2.2 units (3 marks)

$\_^{1}$ finds radius C_1 circle	$\_^{1}$ radius $C_1 = 7.2$
$\_^{2}$ finds distance between centres	$\_^{2}$ $C_1C_2 = 13$
$\_^{3}$ establishes d	$\_^{3}$ $d = (7.2 + 8) - 13 = 2.2$