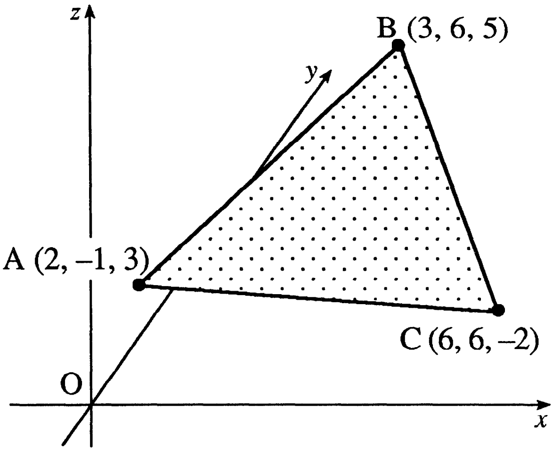
A triangle ABC has vertices A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).



(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	2	3.1			2				3.1.1		Source
(*b*)	5	3.1			5				3.1.11		**1998 Paper 2**
(*c*)	2	0.1			2				0.1		**Qu. 1**

(*a*)

•[1] $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

•[2] $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$

(*b*)

•[3] $\cos B\hat{A}C = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|}$ *stated or implied by responses to •[4] to •[7]*

•[4] $\overrightarrow{AB}.\overrightarrow{AC} = 4 + 49 - 10$

•[5] $\overrightarrow{AB} = \sqrt{54}$

•[6] $\overrightarrow{AC} = \sqrt{90}$

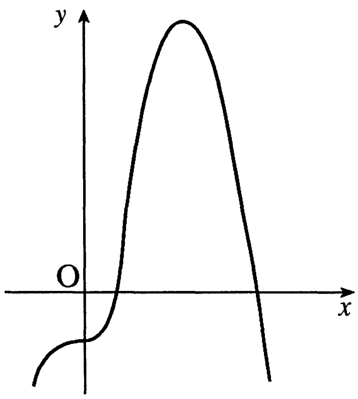
•[7] $B\hat{A}C = 51 \cdot 9°$

(*c*)

•[8] **identify** 2 sides and included angle *e.g.* $\sqrt{54}$, $\sqrt{90}$, $B\hat{A}C$

•[9] $27 \cdot 4$

A curve has equation $y = -x^4 + 4x^3 - 2$. An incomplete sketch of the graph is shown in the diagram.



(a) Find the coordinates of the stationary points. **(6)**

(b) Determine the nature of the stationary points. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(a)	6	1.3					6		1.3.12		Source
(b)	2	1.3					2		1.3.12		**1998 Paper 2 Qu. 2**

(a)

•1 $\frac{dy}{dx} = \ldots\ldots$ *stated or implied* by •2

•2 $-4x^3 + 12x^2$

•3 $-4x^3 + 12x^2 = 0$ *or* $\frac{dy}{dx} = 0$ *explicitly stated*

•4 $-4x^2(x-3)$ (*accept* $x^2(-4x+12)$)

•5 $x = 0$ *and* 3

•6 $y = -2$ *and* 25

(b)

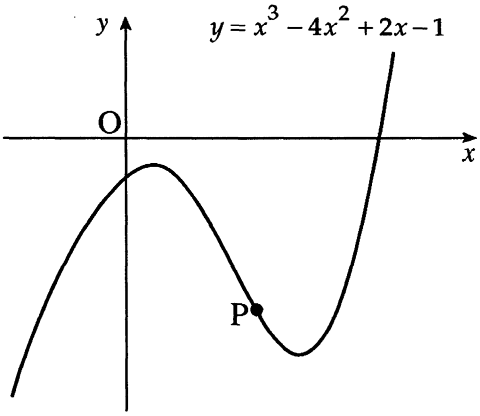
•7

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$					

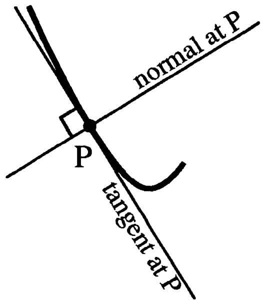
•8 $+$ 0 $+$ 0 $-$

PI at $x = 0$, max *at* $x = 3$

(*a*) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the tangent to the curve at the point P where $x = 2$. **(5)**



(*b*) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P.
Find the angle which the normal at P makes with the positive direction of the x-axis. **(2)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	1.3
(*a*)	5	1.3					5		1.1.7, 1.3.9, 1.1.6	Source **1998 Paper 2**
(*b*)	2	1.1					2		1.1.3, 1.1.9	**Qu. 3**

(*a*)
- •1 $\frac{dy}{dx} = \dots$
- •2 $3x^2 - 8x + 2$
- •3 gradient $= -2$ (calculated from $\frac{dy}{dx}$)
- •4 $y_A = -5$
- •5 $y + 5 = -2(x - 2)$

(*b*)
- •6 $m_{\text{normal}} = \frac{1}{2}$
- •7 angle $= \tan^{-1}\frac{1}{2}$

A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

(*a*) The parabola has equation of the form $y = ax(b - x)$. Determine the values of a and b. **(2)**

(*b*) Find the area enclosed by the parabola and the x-axis. **(4)**

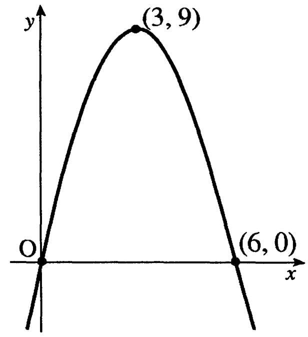


Diagram 1

(*c*) (i) Diagram 2 shows the parabola from (*a*) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

(ii) Calculate the area enclosed between the parabola and the line. **(5)**

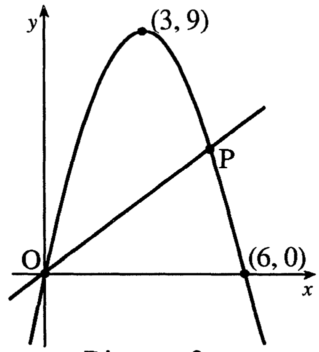


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	2	1.2	2						1.2.7		Source
(*b*)	4	2.2	4						2.2.6		**1998 Paper 2**
(c)i	2	2.1	2						2.1.8		**Qu. 4**
(c)ii	3	2.2		3					2.2.7		

(*a*) •[1] $a = 1$

•[2] $b = 6$

(*b*) •[3] $\int_0^6 x(6-x)\,dx$

•[4] $\int \left(6x - x^2\right)dx$

•[5] $3x^2 - \frac{1}{3}x^3$

•[6] 36

(*c*) •[7] $x = 6x - x^2$

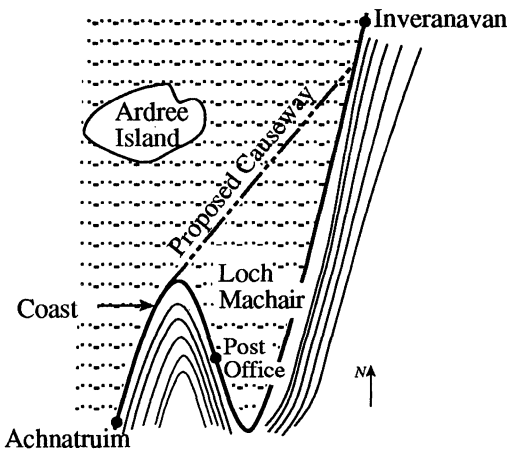
•[8] $x_P = 5$

•[9] $\int_0^5 \left(6x - x^2 - x\right)\,dx$ or equiv.

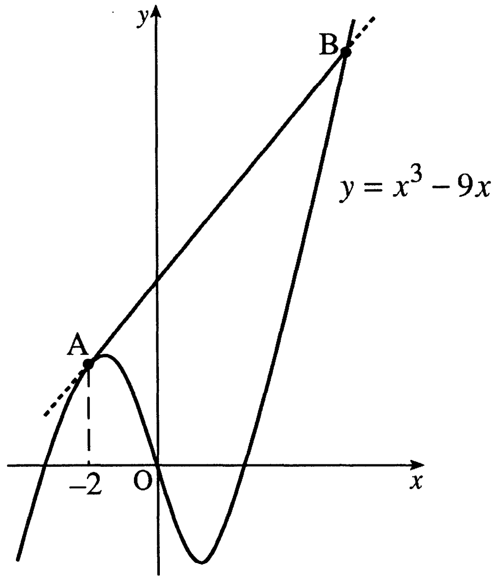
•[10] $\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$ or equiv.

•[11] $\frac{125}{6}$ or equiv.

The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



(*a*) (i) Write down the coordinates of A. **(5)**

(ii) Find the equation of the line AB.

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.1
(*a*)i	1	0.1	1						0.1	Source
(*a*)ii	4	1.1	4						1.1.6, 4	**1998 Paper 2**
(*b*)	7	2.1	2	5					2.1.12 & 2.1.2	**Qu. 5**

(*a*)

•1 $y_{x=-2} = 10$

•2 $\frac{dy}{dx} = \ldots\ldots$

•3 $3x^2 - 9$

•4 $m_{x=-2} = 3$

•5 $y - 10 = 3(x + 2)$

(*b*)

•6 $y = 3x + 16$

•7 $3x + 16 = x^3 - 9x$

•8 $x^3 - 12x - 16 = 0$

•9 *e.g.*

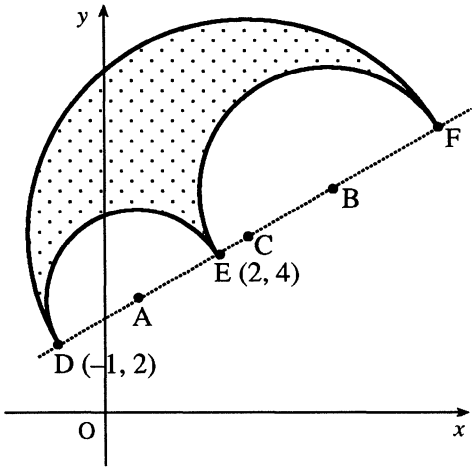
–2	1	0	–12	–16
		–2	4	16
	1	–2	–8	0

•10 *e.g.* $x^2 - 2x - 8$

•11 *e.g.* $(x + 2)(x - 4)$

•12 B is $(4, 28)$

The shape shown in the diagram is composed of 3 semicircles with centres A, B and C which lie on a straight line.



DE is a diameter of one of the semicircles. The coordinates of D and E are (–1, 2) and (2, 4).

(*a*) Find the equation of the circle with centre A and diameter DE. **(3)**

The circle with centre B and diameter EF has equation $x^2 + y^2 - 16x - 16y + 76 = 0$.

(*b*) (i) Write down the coordinates of B.

(ii) Determine the coordinates of F and C. **(3)**

(*c*) In the diagram the perimeter of the shape is represented by the thick black line. Show that the perimeter is $5\pi\sqrt{13}$ units. **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	2.4
			C	A/B	C	A/B	C	A/B	Main Additional	
(*a*)	3	2.4					3		2.4.3	Source
(*b*)	3	2.4					3		2.4.2 & 3.1.6	**1998 Paper 2**
(*c*)	3	0.1						3	0.1	**Qu. 6**

(***a***) •1 $A = \left(\frac{1}{2}, 3\right)$

•2 $r^2 = \frac{9}{4} + 1$ ***or*** $d^2 = 13$

•3 $\left(x - \frac{1}{2}\right)^2 + (y-3)^2 = \frac{13}{4}$

or $x^2 + y^2 - x - 6y + 6 = 0$

(***b***) •4 $B\ (8,8)$

•5 $F\ (14,12)$

•6 $C\ \left(\frac{13}{2}, 7\right)$

(***c***) •7 $\frac{1}{2}\pi DF + \frac{1}{2}\pi DE + \frac{1}{2}\pi EF$

•8 $\frac{1}{2}\pi DF = \frac{5}{2}\pi\sqrt{13}$ ***OR*** $\frac{1}{2}\pi EF = 2\pi\sqrt{13}$

•9 $\frac{5}{2}\pi\sqrt{13} + \frac{1}{2}\pi\sqrt{13} + 2\pi\sqrt{13}$

The function f is defined by $f(x) = 2\cos x° - 3\sin x°$.

(*a*) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x + \alpha)°$ where $k > 0$ and $0 \le \alpha < 360$, and determine the values of k and α. **(4)**

(*b*) Hence find the maximum and minimum values of $f(x)$ and the values of x at which they occur, where x lies in the interval $0 \le x < 360$. **(4)**

(*c*) Write down the minimum value of $(f(x))^2$. **(1)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	3.4
			C	A/B	C	A/B	C	A/B	Main Additional	
(*a*)	4	3.4			4				3.4.1	Source **1998 Paper 2 Qu. 7**
(*b*)	4	3.4			1	3			3.4.3	
(*c*)	1	0.1				1			0.1	

(*a*)

- $\bullet^1$ $k\cos x\cos\alpha - k\sin x\sin\alpha$ **stated explicitly**
- $\bullet^2$ $k\sin\alpha = 3$ *and* $k\cos\alpha = 2$ **stated explicitly**
- $\bullet^3$ $k = \sqrt{13}$
- $\bullet^4$ $\alpha = 56.3$

(*b*)

- $\bullet^5$ $\sqrt{13}\cos(x + 56.3)$
- $\bullet^6$ $\text{Max} = \sqrt{13}$ ***and*** $\text{min} = -\sqrt{13}$
- $\bullet^7$ $x = 303.7$ and no further answers
- $\bullet^8$ $x = 123.7$ and no further answers

(*c*)

- $\bullet^9$ Min Value $= 0$

A gardener feeds her trees weekly with "Bioforce, the wonder plant food".
It is known that in a week the amount of plant food in the tree falls by about 25%.

(*a*) The trees contain no Bioforce initially and the gardener applies 1g of Bioforce to each tree every Saturday. Bioforce is only effective when there is continuously more than 2g of it in the tree. Calculate how many weekly feeds will be necessary before the Bioforce becomes effective. **(3)**

(*b*) (i) Write down a recurrence relation for the amount of plant food in the tree immediately after feeding. **(1)**

(ii) If the level of Bioforce in the tree exceeds 5g, it will cause leaf burn. Is it safe to continue feeding the trees at this rate indefinitely? **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.4
(*a*)	3	1.4			3				1.4.1	Source
(*b*)	1	1.4			1				1.4.3	**1998 Paper 2**
(*c*)	4	1.4			4				1.4.4, 1.4.5	**Qu. 8**

(*a*) •1 75% or equivalent

•2 0.75, 1.31 **and** 1.73

•3 2.05 **and** "after fourth feed"

(*b*) •4 $u_{n+1} = 0.75u_n + 1$

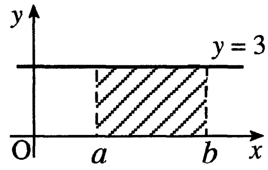
(*c*) •5 $-1 < 0.75 < 1$ so sequence has a limit

•6 *e.g.* $L = 0.75L + 1$

•7 $L = 4$

•8 Safe to continue

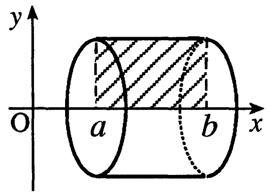
Diagram 1 shows the area between the line $y = 3$ and the x-axis from $x = a$ to $x = b$. If this area is rotated through 360° about the x-axis, it forms a solid shape (a cylinder) as shown in Diagram 2.



Diag.1

The volume of this solid may be obtained by

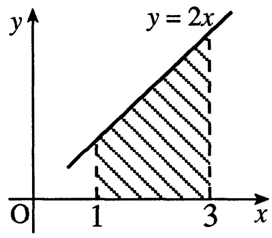
evaluating the integral $\pi \int_a^b y^2 \, dx.$



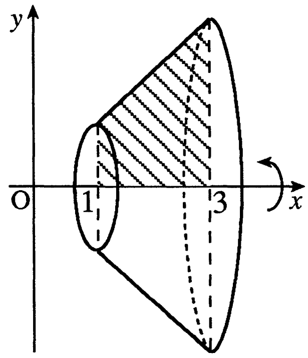
Diag.2

Worked Example

The area between $y = 2x$ and the x-axis from $x = 1$ to $x = 3$ is rotated about the x-axis. The volume of the solid is calculated as follows:

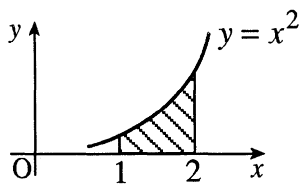


$$y = 2x$$
$$y^2 = (2x)^2 = 4x^2$$

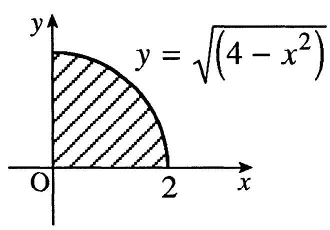


$$\pi \int_1^3 y^2 \, dx$$
$$= \pi \int_1^3 4x^2 \, dx$$
$$= \pi \left[\tfrac{4}{3} x^3\right]_1^3$$
$$= \pi \left[36 - \tfrac{4}{3}\right]$$
$$\text{Volume } = \tfrac{104}{3} \pi \text{ units}^3$$

(*a*) Use this method to find the volume of the solid formed when the area between $y = x^2$ and the x-axis from $x = 1$ to $x = 2$ is rotated about the x-axis. **(4)**



(*b*) (i) Use this method to find the volume of the solid formed when the area between $y = \sqrt{(4 - x^2)}$ and the x-axis from $x = 0$ to $x = 2$ is rotated about the x-axis. **(4)**



(ii) **Hence** write down the volume of a sphere of radius 2. **(1)**

1998 Paper 2 Qu. 9

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	4
			C	A/B	C	A/B	C	A/B	Main Additional	
(*a*)	4	2.2					4		2.2.5	Source **1998 Paper 2 Qu. 9**
(*b*)	4	2.2					4		2.2.5	
(*c*)	1	0.1						1	0.1	

(*a*) •1 $y^2 = x^4$

•2 $\pi\int_1^2 x^4 \, dx$

•3 $\pi\left[\frac{1}{5}x^5\right]_1^2$

•4 $\frac{31}{5}\pi$ (*accept* 19.5)

(*b*) •5 $y^2 = 4 - x^2$

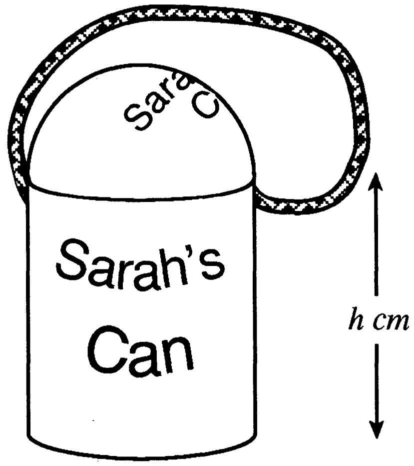
•6 $\pi\int_0^2 \left(4 - x^2\right) \, dx$

•7 $\pi\left[4x - \frac{1}{3}x^3\right]_0^2$

•8 $\frac{16}{3}\pi$

(*c*) •9 $\frac{32}{3}\pi$ **or** $2\times\frac{16}{3}\pi$

A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.



(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \dfrac{800}{r}$. **(3)**

Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	3	0.1				3			0.1		Source **1998 Paper 2 Qu. 10**
(*b*)	6	1.3			3	3			1.3.15		

(*a*)
- •1 $\pi r^2 + 2\pi rh + 2\pi r^2$
- •2 $h = \frac{400}{\pi r^2}$ or equivalent (*e.g.* $\pi rh = \frac{400}{r}$)
- •3 $2\pi r \frac{400}{\pi r^2} + 3\pi r^2$ and completes proof

(*b*)
- •4 $\frac{dA}{dr} = \ldots$
- •5 $800r^{-1}$
- •6 $6\pi r - 800r^{-2}$
- •7 *e.g.* $6\pi r - \frac{800}{r^2} = 0$
- •8 3.5
- •9

r	3.5^-	3.5	3.5^+
$\frac{dA}{dr}$	$-ve$	0	$+ve$

(*a*) The variables x and y are connected by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log_e y$ and x. **(3)**

(*b*) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

x	3.1	3.5	4.1	5.2
y	21 876	72 631	439 392	11 913 076

The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b. **(6)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	3	3.3				3			3.3.7		Source
(*b*)	6	3.3				6			3.3.5		**1998 Paper 2 Qu. 11**

(*a*)
- •1 $\log_e y = \log_e ae^{bx}$
- •2 $\log_e y = \log_e a + \log_e e^{bx}$
- •3 $\log_e y = \log_e a + bx$

(*b*)
- •4 evidence for strategy being carried out will be appearance of two equations at •5 stage
- •5 *e.g.* $3.1b + \log a = 9.99,\ 5.2b + \log a = 16.29$
- •6 strategy: know to subtract
- •7 $b = 3$
- •8 $a = e^{0.69}$
- •9 $a = 2$

Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1, with equation $y = 2x + 1$.
The line intersects the circle at A and B.

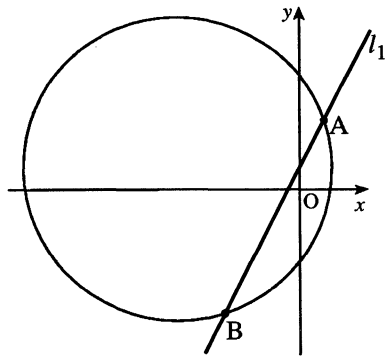


Diagram 1

(*a*) Find the coordinates of the points A and B. **(5)**

(*b*) Diagram 2 shows a second line, l_2, which passes through the centre of the circle, C, and is at right angles to line l_1.

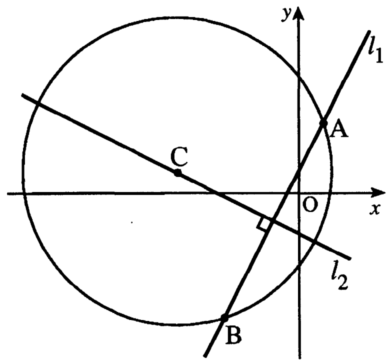


Diagram 2

(i) Write down the coordinates of C. **(1)**

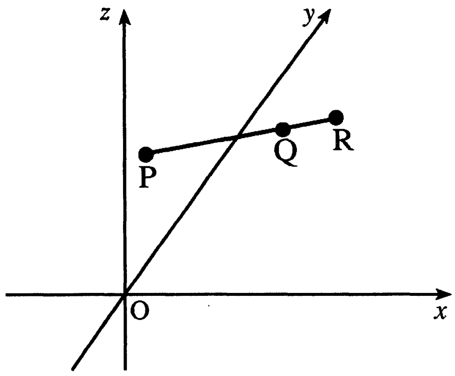
(ii) Find the equation of the line l_2. **(3)**

part marks	Unit	non-calc		calc		calc neut		Content Reference :		2.4
		C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*) 5	2.4					5		2.4.4		Source
(*b*)i 1	2.4					1		2.4.2		**1997 Paper 2**
(*b*)ii 3	1.1					3		1.1.10	1.1.7	**Qu.1**

(*a*)
- •[1] know to substitute
- •[2] correct substitution
- •[3] a "quadratic" = 0
- •[4] $x = -3,\ 1$
- •[5] $y = -5,\ 3$

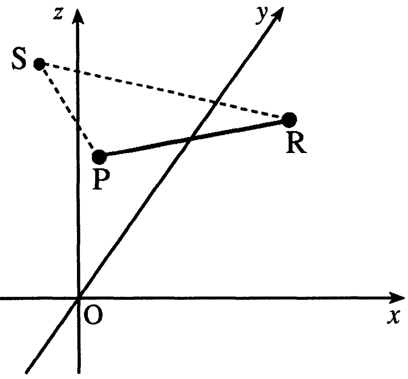
(*b*)
- •[6] $m_{diameter} = 2$
- •[7] $m_{perpendicular} = -\frac{1}{2}$
- •[8] centre = $(-1, -1)$
- •[9] equation: $y + 1 = -\frac{1}{2}(x + 1)$

Relative to the axes shown and with an appropriate scale, P(–1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



(*a*) Find the coordinates of R. **(3)**

(*b*) Roads from P and R are built to meet at the point S (–2, 2, 5). Calculate the size of angle PSR. **(7)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	3	3.1			3				3.1.6		Source
(*b*)	7	3.1			7				3.1.11		**1997 Paper 2 Qu.2**

(*a*)

•1 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ •2 $\begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$

•3 R = (7, –1, 6)

(*b*)

•4 $\overrightarrow{SP}.\overrightarrow{SR} = |SP||SR|\cos P\hat{S}R$

•5 $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ •6 $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$

•7 $|SP| = \sqrt{11}$ •8 $|SR| = \sqrt{91}$

•9 $\overrightarrow{SP}.\overrightarrow{SR} = 3$

•10 $P\hat{S}R = 84 \cdot 6°$

The sum of £1000 is placed in an investment account on January 1st and, thereafter, £100 is placed in the account on the first day of each month.

- Interest at the rate of 0·5% per month is credited to the account on the last day of each month.
- This interest is calculated on the amount in the account on the first day of the month.

(*a*) How much is in the account on June 30th ? **(4)**

(*b*) On what date does the account first exceed £2000? **(2)**

(*c*) Find a recurrence relation which describes the amount in the account, explaining your notation carefully. **(3)**

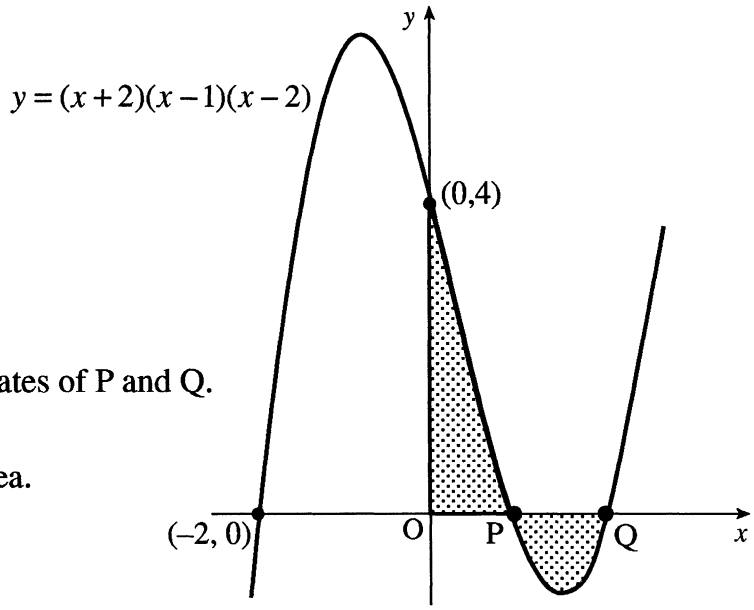
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	4	1.4			4				1.4.1		Source
(*b*)	2	1.4			2				1.4.1		**1997 Paper 2**
(*c*)	3	1.4			3				1.4.3		**Qu.3**

(*a*)
- •1 1.005
- •2 £1000 + interest = £1005
- •3 £1005 + £100 + interest = £1110.525
- •4 £1537.93

(*b*)
- •5 complete another month
- •6 £2073.94 on Nov.1st

(c)
- •7 $u_{n+1} = 1.005u_n + 100$
- •8 u_n = amount on 1st day of each month
- •9 $u_0 = 1000$ (on 1st January)

The diagram shows a sketch of the graph of $y = (x+2)(x-1)(x-2)$. The graph cuts the axes at (–2, 0), (0, 4) and the points P and Q.



(*a*) Write down the coordinates of P and Q. **(2)**

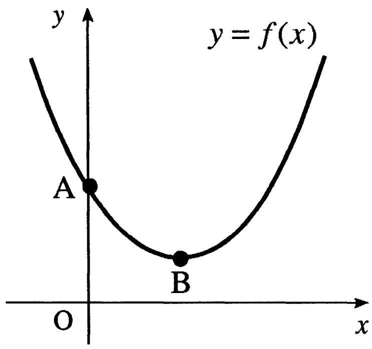
(*b*) Find the total shaded area. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	2	2.1	2						2.1.2		Source
(*b*)	7	2.2	6	1					2.2.6		**1997 Paper 2 Qu.4**

(*a*)
- •[1] (1,0)
- •[2] (2,0)

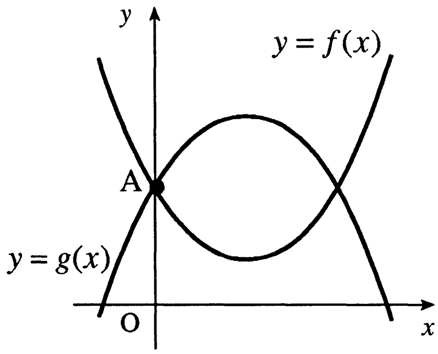
(*b*)
- •[3] $\int f(x)dx$
- •[4] $\int_0^1 - \int_1^2$
- •[5] $(x+2)(x^2-3x+2)$ or equiv.
- •[6] $x^3 - x^2 - 4x + 4$
- •[7] $\frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x$
- •[8] $1\frac{11}{12}$ or $-\frac{7}{12}$
- •[9] $2\frac{1}{2}$

The first diagram shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x-2)^2 + 1$.
The graph cuts the y-axis at A and has a minimum turning point at B.



(*a*) Write down the coordinates of A and B. **(3)**

(*b*) The second diagram shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.
Find the area enclosed by the two curves. **(5)**



(*c*) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants.
Write down the values of m and n. **(2)**

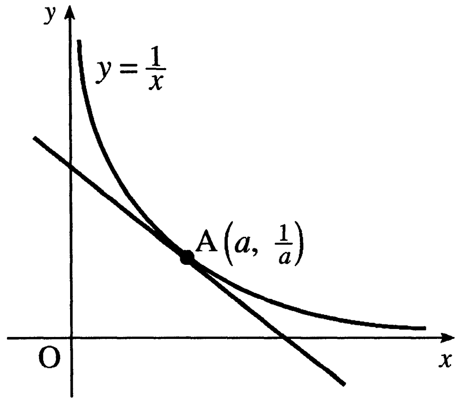
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	3	1.2	3						1.2.9		Source
(*b*)	5	2.2	5						2.2.7		**1997 Paper 2**
(*c*)	3	0.1		2					0.1		**Qu.5**

(*a*)
- •1 $A = (0,5)$
- •2 $x_B = 2$
- •3 $y_B = 1$

(*b*)
- •4 $\int_0^4$
- •5 $\int\left(\left(5+4x-x^2\right)-\left(x^2-4x+5\right)\right)\,dx$
- •6 $8x - 2x^2$ *or equiv.*
- •7 $4x^2 - \frac{2}{3}x^3$ *or equiv.*
- •8 $\frac{64}{3}$

(*c*)
- •9 $n = -1$
- •10 $m = 10$

(*a*) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram. The tangent at A $\left(a, \frac{1}{a}\right)$ has been drawn.
Find the gradient of this tangent. **(4)**



(*b*) Hence show that the equation of this tangent is $x + a^2 y = 2a$. **(2)**

(*c*) This tangent cuts the y-axis at B and the x-axis at C.

(i) Calculate the area of triangle OBC **(3)**

(ii) Comment on your answer to c(i). **(1)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	1.3
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(*a*)	4	1.3					4		1.3.7	**1997 Paper 2**
(*b*)	2	1.1					1	1	1.1.7	**Qu.6**
(*c*)	4	0.1						4	0.1	

(a)
- •1 $\frac{1}{x} = x^{-1}$
- •2 $\frac{dy}{dx} = \ldots\ldots$
- •3 $\frac{dy}{dx} = -x^{-2}$
- •4 gradient = $-a^{-2}$

(b)
- •5 use $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$
- •6 $a^2 y - a = -(x - a)$ and completes proof

(c)
- •7 $y_B = \frac{2a}{a^2}$
- •8 $x_A = 2a$
- •9 2
- •10 independent of a

In certain topics in Mathematics, such as calculus, we often require to write an expression such as $\frac{8x+1}{(2x+1)(x-1)}$ in the form $\frac{2}{2x+1}+\frac{3}{x-1}$.

$\frac{2}{2x+1}+\frac{3}{x-1}$ are called **Partial Fractions** for $\frac{8x+1}{(2x+1)(x-1)}$.

The worked example shows you how to find partial fractions for the expression $\frac{6x+2}{(x+2)(x-3)}$.

Worked Example

Find partial fractions for $\frac{6x+2}{(x+2)(x-3)}$.

$$\text{Let } \frac{6x+2}{(x+2)(x-3)} = \frac{A}{x+2}+\frac{B}{x-3} \quad \text{where } A \text{ and } B \text{ are constants}$$

$$= \frac{A(x-3)}{(x+2)(x-3)}+\frac{B(x+2)}{(x-3)(x+2)}$$

$$\text{i.e. } \frac{6x+2}{(x+2)(x-3)} = \frac{A(x-3)+B(x+2)}{(x+2)(x-3)}$$

Hence $6x+2 = A(x-3)+B(x+2)$ for all values of x.

A and B can be found as follows:

Select a value of x that makes the first bracket zero

Let $x = 3$ (this eliminates A)

$18+2 = A\times 0 \ + \ B\times 5$

$20 = 5B$

$\underline{B = 4}$

Select a value of x that makes the second bracket zero

Let $x = -2$ (this eliminates B)

$-12+2 = A\times(-5) \ + \ B\times 0$

$-10 = -5A$

$\underline{A = 2}$

$$\text{Therefore } \frac{6x+2}{(x+2)(x-3)} = \frac{2}{x+2}+\frac{4}{x-3}.$$

Find partial fractions for $\frac{5x+1}{(x-4)(x+3)}$. **(6)**

1997 Paper 2 Qu.7

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	4
			C	A/B	C	A/B	C	A/B	Main Additional	Source
	6	0.1					6		0.1	1997 Paper 2 Qu.7

•1 $\frac{A}{x-4}+\frac{B}{x+3}$

•2 $\frac{A(x+3)+B(x-4)}{(2x-1)(x+3)}$

•3 $5x+1=A(x+3)+B(x-4)$

•4 choose to let $x=-3$ and 4 in turn

•5 $A=3$

•6 $B=2$

The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal, and seeds.
Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

(*a*) The half-life of carbon-14, i.e. the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k, correct to 3 significant figures. **(3)**

(*b*) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	3	3.3			1	2			3.3.7		Source
(*b*)	3	3.3				3			3.3.7		**1997 Paper 2 Qu.8**

(*a*) •1 $\frac{1}{2}y_0 = y_0 e^{5700k}$

•2 $\ln\frac{1}{2} = 5700k$

•3 $k = -0.000122$

(*b*) •4 $y = y_0 e^{-0.000122 \times 1000}$

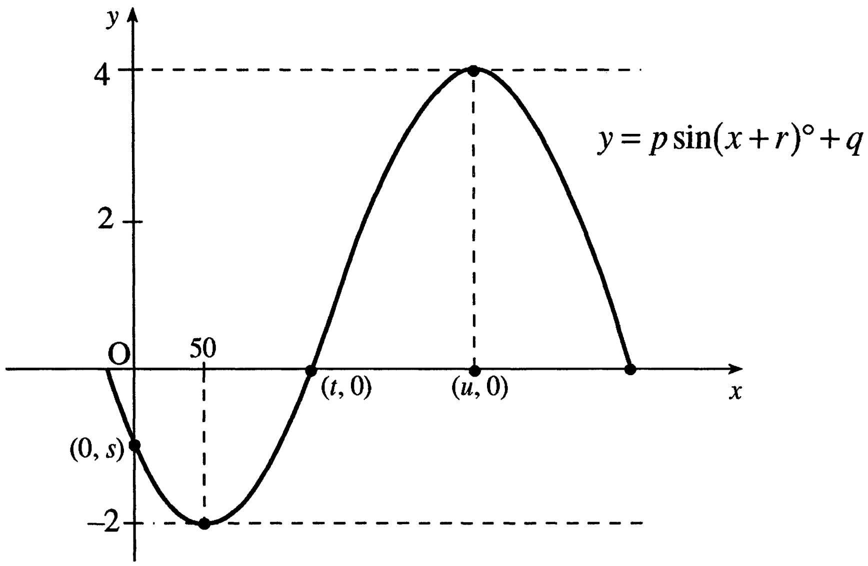
•5 $\frac{y}{y_0} = \ldots$

•6 88.5%

The sketch represents part of the graph of a trigonometric function of the form $y = p\sin(x+r)° + q$. It crosses the axes at (0, s) and (t,0), and has turning points at (50, –2) and (u, 4).

(i) Write down values for p, q, r and u. **(4)**

(ii) Find the values for s and t. **(4)**

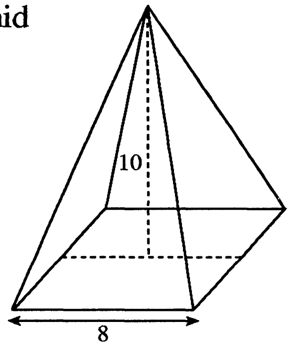


part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(*a*)	4	1.2			2	2			1.2.3		Source
(*b*)	4	2.3				4			2.3.1		**1997 Paper 2 Qu.9**

(*a*)
- •[1] $p = -3$
- •[2] $q = 1$
- •[3] $r = 40$ *or* -320
- •[4] $u = 230$

(*b*)
- •[5] replace x by 0
- •[6] –0.928
- •[7] replace y by 0
- •[8] 120.5

A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm. and a vertical height of 10cm.

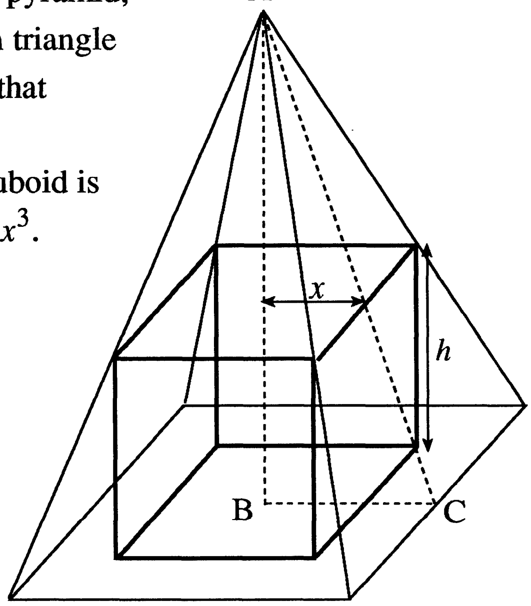


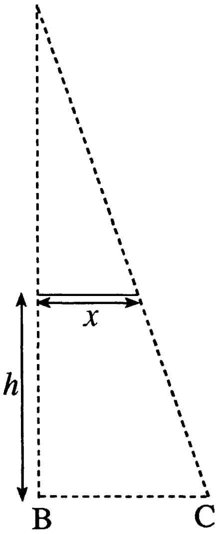
(*a*) The cuboid has a square base of side $2x$ cm and a height of h cm.

If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that

(i) $h = 10 - \frac{5}{2}x$. **(3)**

(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$. **(1)**





(*b*) Hence find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	4	0.1					1	3	0.1		Source
(*b*)	6	1.3					3	3	1.3.15		**1997 Paper 2 Qu.10**

(*a*)
- •1 strategy: e.g. equate ratios from similar triangles
- •2 $\frac{10}{4} = \frac{10-h}{x}$ or equivalent
- •3 complete proof
- •4 $V = 40x^2 - 10x^3$

(*b*)
- •5 $\frac{dV}{dx} =$
- •6 $80x - 30x^2$
- •7 $\frac{dV}{dx} = 0$ for stationary points
- •8 $0, \frac{8}{3}$
- •9

x	…	$\frac{8}{3}$	…
$\frac{dV}{dx}$	+	0	−
		max	

- •10 $\frac{16}{3}$ and $\frac{10}{3}$

Two identical coins, radius 1 unit, are supported by horizontal and vertical plates at B and C. Diagram 1 shows the coins touching each other and the line of centres is inclined at p radians to the vertical.

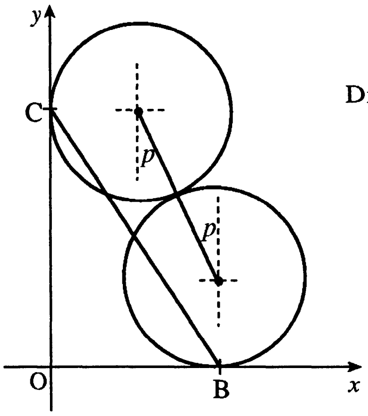


Diagram 1

Let d be the length of BC.

(*a*) (i) Show that $\text{OB} = 1 + 2\sin p$ **(1)**

(ii) Write down a similar expression for OC and hence show that $d^2 = 6 + 4\cos p + 4\sin p$. **(2)**

(*b*) (i) Express d^2 in the form $6 + k\cos(p - \alpha)$ **(4)**

(ii) Hence write down the exact maximum value of d^2 and the value of p for which this occurs. **(2)**

(*c*) Diagram 2 shows the special case where $p = \frac{\pi}{4}$.

(i) Show that $\text{OB} = 1 + \sqrt{2}$ and find the exact length of BD. **(2)**

(ii) Using your answer to (*b*)(ii) find the exact value of $\sqrt{6 + 4\sqrt{2}}$. **(2)**

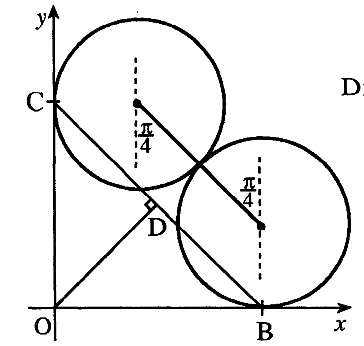


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.4
(*a*)	3	0.1	2	1					0.1		Source
(*b*)	6	3.4	4	2					3.4.1	3.4.3	**1997 Paper 2**
(*c*)	4	0.1	1	3					0.1		**Qu.11**

(*a*)

- •1 $\sin p = \frac{\text{"hor"}}{2}$ *and* $\text{OB} = 1 + \text{"hor"}$
- •2 $\text{OC} = 1 + 2\cos p$
- •3 $d^2 = (1 + 2\cos p)^2 + (1 + 2\sin p)^2$ and completes proof

(*b*)

- •4 $k\cos(p - \alpha) = k\cos p\cos\alpha + k\sin p\sin\alpha$
- •5 $k\cos\alpha = 4$ *and* $k\sin\alpha = 4$
- •6 $k = 4\sqrt{2}$
- •7 $\alpha = \frac{\pi}{4}$
- •8 maximum value $= 6 + 4\sqrt{2}$
- •9 occurs when $p = \frac{\pi}{4}$

(*c*)

- •10 $OB = 1 + 2\sin\frac{\pi}{4}$ and completes proof
- •11 $BD = (1 + \sqrt{2}) \times \frac{1}{\sqrt{2}}$
- •12 $\text{BC} = 2 + \sqrt{2}$
- •13 $6 + 4\sqrt{2} = (2 + \sqrt{2})^2$ *so* $\sqrt{6 + 4\sqrt{2}} = 2 + \sqrt{2}$

A curve has equation $y = x^4 - 4x^3 + 3$.

(a) Find algebraically the coordinates of the stationary points. **(6)**

(b) Determine the nature of the stationary points. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(a)	6	1.3	6						1.3.12		Source **1996 Paper 2 Qu.1**
(b)	2	1.3	2						1.3.12		

(a)

- •1 $\frac{dy}{dx} =$
- •2 $4x^3 - 12x^2$
- •3 $= 0$ stated explicitly
- •4 *e.g.* $4x^2(x-3)$
- •5 $x = 0,\ 3$
- •6 $y = 3,\ -24$

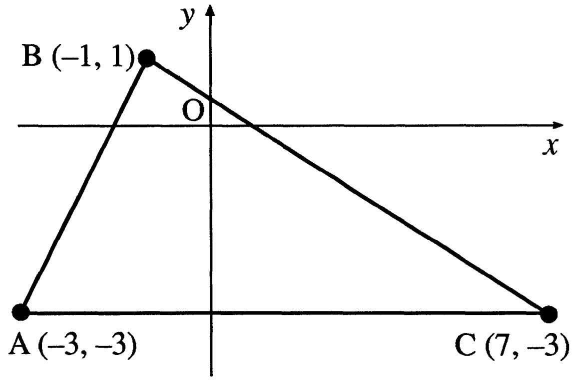
(b)

- •7

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$	$-$	0	$-$	0	$+$

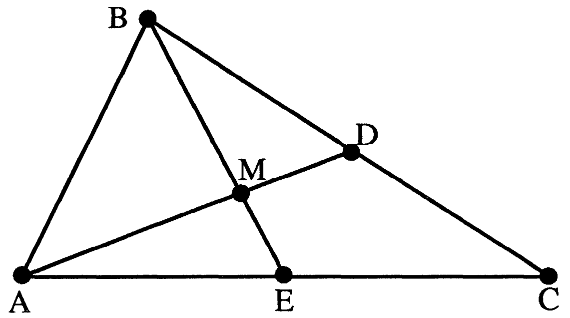
- •8 pt of inflection at $x = 0$
  minimum at $x = 3$

A triangle ABC has vertices A(–3, –3), B(–1, 1) and C(7, –3).



(*a*) Show that the triangle ABC is right-angled at B. **(3)**

(*b*) The medians AD and BE intersect at M.



(i) Find the equations of AD and BE. **(5)**

(ii) Hence find the coordinates of M. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.1
(*a*)	3	1.1					3		1.1.10	Source
(*b*)i	5	1.1					5		1.1.10	**1996 Paper 2**
(b)ii	3	0.1					3		0.1	**Qu.2**

(*a*)
- •1 $m_{AB} = 2$
- •2 $m_{BC} = -\frac{1}{2}$
- •3 $m_{AB} \times m_{BC} = -1 \Rightarrow m_{AB} \perp m_{BC}$

(*b*)
- •4 $D = (3,-1)$ *and* $E = (2,-3)$
- •5 $m_{AD} = \frac{1}{3}$
- •6 AD: $y+1 = \frac{1}{3}(x-3)$ *or equiv.*
- •7 $m_{BE} = -\frac{4}{3}$
- •8 BE: $y-1 = -\frac{4}{3}(x+1)$ *or equiv.*
- •9 *eg* clear fractions
- •10 *eg* substitute
- •11 $x = 1,\ y = -\frac{5}{3}$

The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

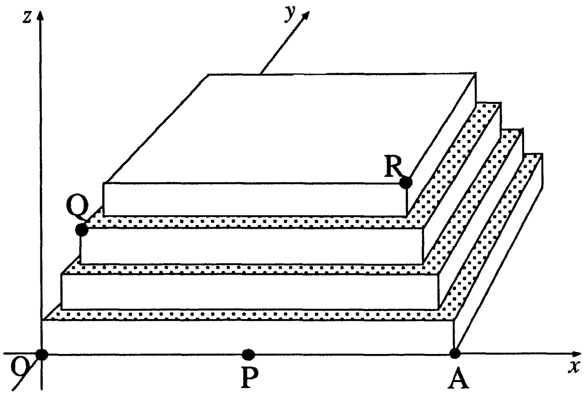


Diagram 1

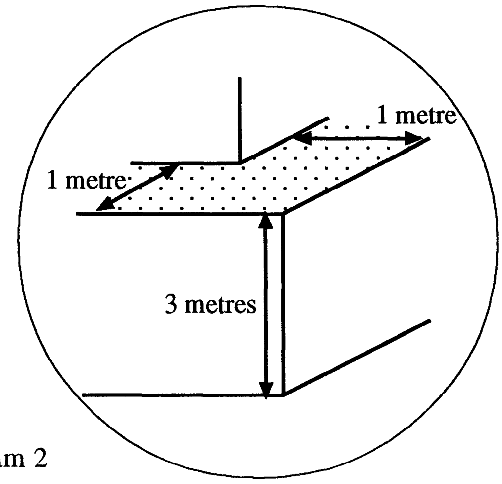


Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(*a*) Find the coordinates of Q and R. **(2)**
(*b*) Find the size of angle QPR. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	2	3.1			2				3.1.1		Source
(*b*)	7	3.1			7				3.1.11		**1996 Paper 2 Qu.3**

(*a*) •1 $Q=(2,2,9)$

•2 $R=(21,3,12)$

(*b*) •3 $\cos\theta=\frac{a.b}{|a||b|}$ *with some subsequent use*

eg $\cos Q\hat{P}R=\frac{\overrightarrow{PQ}.\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}$

•4 $\overrightarrow{PQ}=\begin{pmatrix}-10\\2\\9\end{pmatrix}$ •5 $\overrightarrow{PR}=\begin{pmatrix}9\\3\\12\end{pmatrix}$

•6 $|\overrightarrow{PQ}|=\sqrt{185}$

•7 $|\overrightarrow{PR}|=\sqrt{234}$

•8 $\overrightarrow{PQ}.\overrightarrow{PR}=24$

•9 $Q\hat{P}R=83\cdot4^\circ$

(*a*) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. **(2)**

(ii) Find $f(g(x))$. **(2)**

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. **(2)**

(ii) Determine the nature of the roots of this equation when $k = 6$. **(2)**

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	4	1.2	4						1.2.6		Source
(*b*)	7	2.1	7						2.1.6,	2.1.7, 0.1	**1996 Paper 2 Qu.4**

(*a*)

•1 $g(2x+1)$

•2 $(2x+1)^2 + k$

•3 $f(x^2 + k)$

•4 $2(x^2 + k) + 1$

(*b*)

•5 $4x^2 + 4x + k + 1$ *AND* $2x^2 + 2k + 1$

•6 $4x^2 + 4x + k + 1 - (2x^2 + 2k + 1) = 0$

so $2x^2 + 4x - k = 0$

•7 $b^2 - 4ac = 16 - 4 \times 2 \times (-6) = 64$

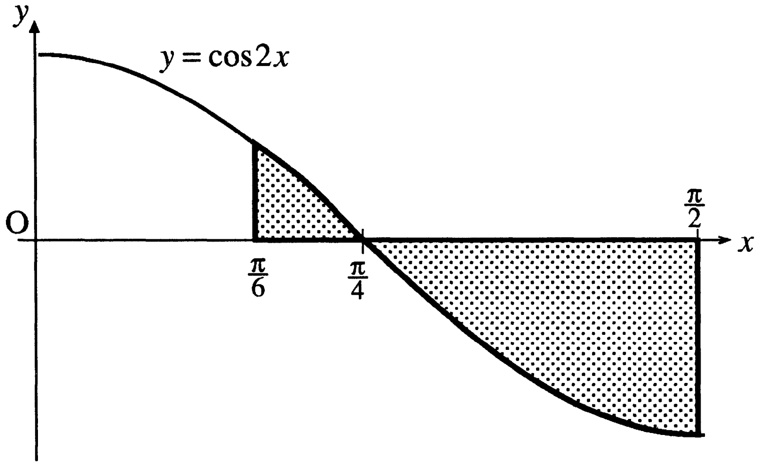
•8 so roots real & distinct

•9 $b^2 - 4ac = 16 - 4 \times 2 \times (-k)$

•10 $b^2 - 4ac = 0$ for equal roots

•11 $k = -2$

An artist has designed a 'bow' shape which he finds can be modelled by the shaded area below. Calculate the area of this shape. **(6)**



part	marks	Unit	non-calc		calc		calc neut		Content Reference :	3.2
			C	A/B	C	A/B	C	A/B	Main Additional	
-	6	3.2	2	4					3.2.4, 2.2.6	Source **1996 Paper 2 Qu.5**

(-) •[1] evidence of two integrals

•[2] $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x \; dx$ *and* $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \; dx$

•[3] $\frac{1}{2}\sin 2x$

•[4] $\frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{2}\sin\frac{\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{4}$

•[5] $\frac{1}{2}\sin\pi - \frac{1}{2}\sin\frac{\pi}{2} = -\frac{1}{2}$

•[6] $1 - \frac{\sqrt{3}}{4}$

Diagram 1 shows :

- the point A(1, 2),
- the straight line $\boldsymbol{l}$ passing through the origin O and the point A.
- the parabola $\boldsymbol{p}$ with a minimum turning point at O and passing through A.
- and the circle $\boldsymbol{c}$, centre O, passing through A.

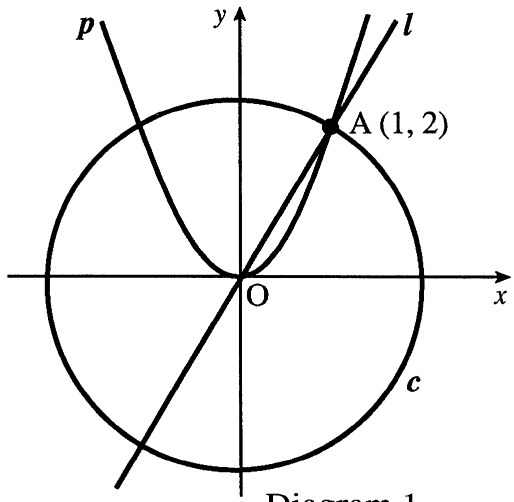


Diagram 1

(a) Write down the equations of the line, the parabola and the circle. **(3)**

The following transformations are carried out:

- the line is given a translation of 4 units down (i.e. – 4 units in the direction of the y-axis).
  Diagram 2 shows the line $\boldsymbol{l'}$, the image of line $\boldsymbol{l}$, after this translation.
- the parabola is reflected in the x-axis.
- the circle is given a translation of 2 units to the right (i.e. +2 units in the direction of the x-axis).

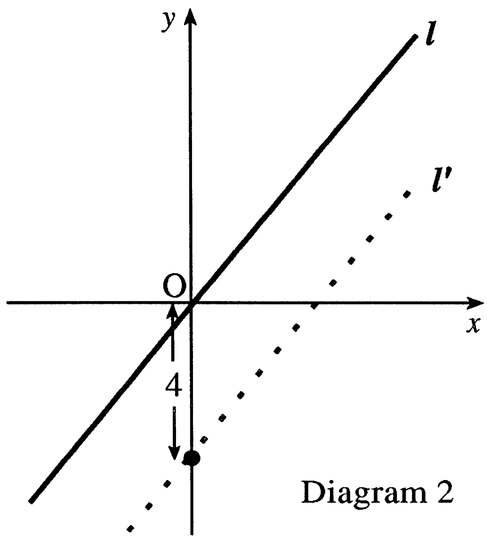


Diagram 2

(b) Write down the equations of $\boldsymbol{l'}$, $\boldsymbol{p'}$ (the image of the parabola p) and $\boldsymbol{c'}$ (the image of the circle c). **(4)**

(c) (i) Show that the line $\boldsymbol{l'}$ passes through the centre of the circle $\boldsymbol{c'}$. **(1)**

(ii) Find the coordinates of the points where the line $\boldsymbol{l'}$ intersects the parabola $\boldsymbol{p'}$. **(3)**

1996 Paper 2 Qu.6

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	2.1
			C	A/B	C	A/B	C	A/B	Main Additional	
(*a*)	3	1.2					3		1.2.7	Source
(*b*)	4	0.1					4		0.1	**1996 Paper 2**
(*c*)	4	2.1					4		2.1.8, 0.1	**Qu.6**

(a) •1 $y = 2x$

•2 $y = 2x^2$

•3 $x^2 + y^2 = 5$

(b) •4 $y = 2x - 4$

•5 $y = -2x^2$

•6 $centre = (2,0)$

•7 $(x-2)^2 + y^2 = 5$

(c) •8 *show* $(2,0)$ *satisfies* $y = 2x - 4$

•9 $2x - 4 = -2x^2$

•10 $(x+2)(x-1) = 0$

•11 $(-2,-8),\ (1,-2)$

$f(x) = 2\cos x° + 3\sin x°$.

(a) Express $f(x)$ in the form $k\cos(x-\alpha)°$ where $k>0$ and $0 \le \alpha < 360$. **(4)**

(b) Hence solve algebraically $f(x) = 0.5$ for $0 \le x < 360$. **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(a)	4	3.4			4				3.4.1		**1996 Paper 2 Qu.7**
(b)	3	3.4			3				3.4.2		

(a)

•1 $k\cos x\cos\alpha + k\sin x\sin\alpha$

•2 $k\cos\alpha = 2$ ***and*** $k\sin\alpha = 3$

•3 $k = \sqrt{13}$

•4 $\alpha = 56.3$

(b)

•5 $\cos(x-56.3)° = \frac{0.5}{\sqrt{13}}$

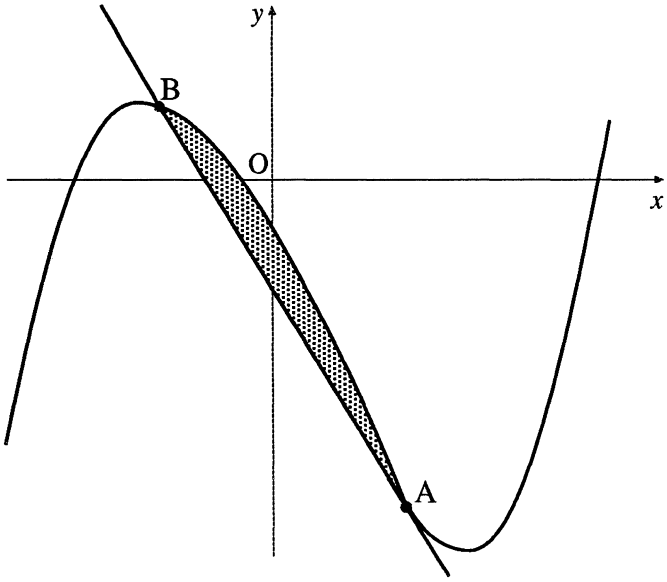
•6 $x - 56.3 = 82.0,\ 278.0$

•7 $x = 138.3,\ 334.3$

In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.2
(*a*)	8	2.1					5	3	2.1.8, 1.1.7, 1.3.9	Source **1996 Paper 2 Qu.8**
(*b*)	3	2.2						3	2.2.7	

(*a*)

- •1 *strat:* $\frac{dy}{dx} = \ldots$
- •2 $\frac{dy}{dx} = 3x^2 - 2x - 6$
- •3 $m_{tgt} = -5$
- •4 $y + 8 = -5(x - 1)$
- •5 *strat:* attempt to simplify and equate y's
- •6 $x^3 - x^2 - x + 1 = 0$
- •7 *strat:* *e.g.* try to factorise
- •8 $B = (-1, 2)$

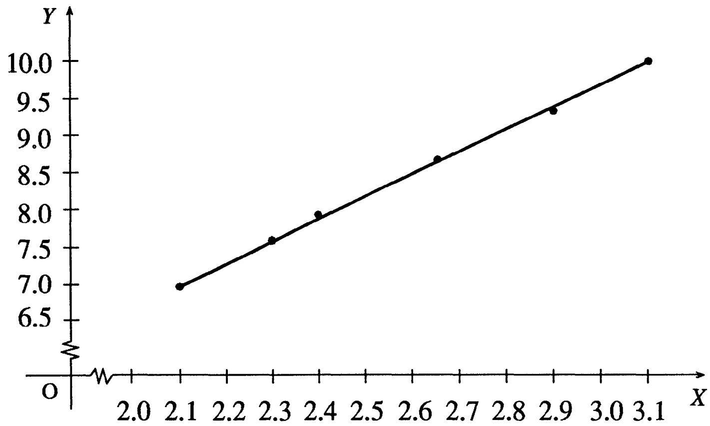
(*b*)

- •9 $\int \left(x^3 - x^2 - 6x - 2\right) - (-5x - 3)\ dx$
- •10 $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$
- •11 $1\frac{1}{3}$

Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

X (=$\log_e x$)	Y (=$\log_e y$)
2.10	7.00
2.31	7.60
2.40	7.92
2.65	8.70
2.90	9.38
3.10	10.00



A graph was drawn and is shown above.

(a) Find the equation of the line in the form $Y = mX + c$. **(3)**

(b) Hence find the values of the constants a and b in the relationship $y = ax^b$. **(4)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	3.3
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(a)	3	1.1			3				1.1.7	**1996 Paper 2 Qu.9**
(b)	4	3.3				4			3.3.6	

(a) $\bullet^1$ *e.g.* $m = 3$

$\bullet^2$ *e.g.* $8.70 = 3 \times 2.65 + c$ *or equiv.*

$\bullet^3$ *e.g.* $Y = 3X + 0.75$

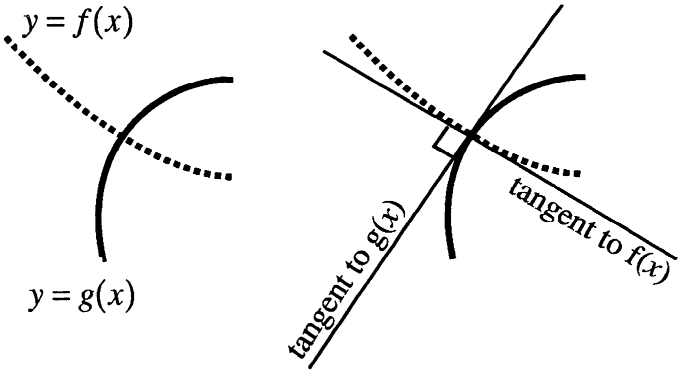
(b) $\bullet^4$ $\ln y = 3\ln x + 0{\cdot}7$

$\bullet^5$ $\ln y = \ln 2.01x^3$

$\bullet^6$ $b = 3$

$\bullet^7$ $a = 2.01$

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



(*a*) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y-5)^2 = 13$.
These two curves intersect at (3, 7) and (–3, 7).
Prove that these curves are orthogonal. **(6)**

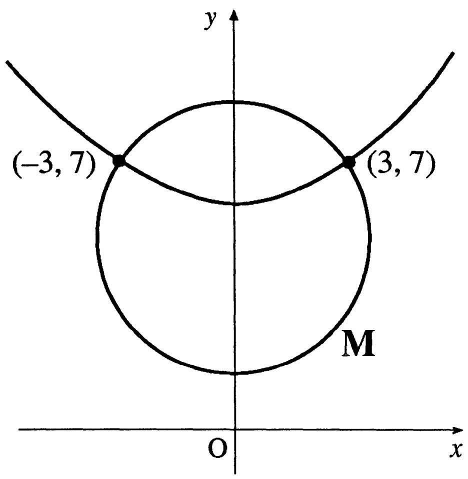


Diagram 1

(*b*) Diagram 2 shows the circle M, from (*a*) above, which is orthogonal to the circle N. The circles intersect at (3, 7) and (–3, 7).

(i) Write down the equation of the tangent to circle M at the point (–3, 7). **(1)**

(ii) Hence find the equation of circle N. **(3)**

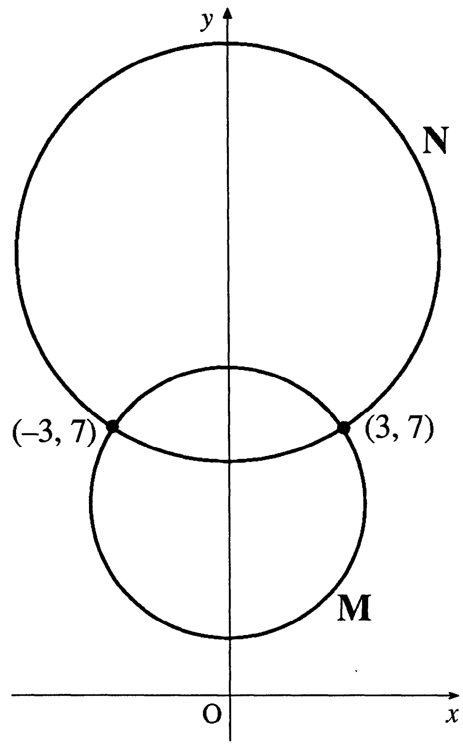


Diagram 2

1996 Paper 2 Qu.10

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	2.4
			C	A/B	C	A/B	C	A/B	Main Additional	Source
(*a*)	6	2.4					3	3	2.4.1, 1.1.9, 1.3.9	**1996 Paper 2**
(*b*)	4	2.4						4	2.4.4	**Qu.10**

(*a*)

•1 $\frac{dy}{dx} = \frac{2}{9}x$

choose 1 point e.g. (3,7)

•2 *parabola*: $m_{tgt\ x=3} = \frac{2}{3}$

•3 circle centre $= (0,5)$

•4 *circle*: $m_{rad\ x=3} = \frac{2}{3}$, $m_{tgt\ x=3} = -\frac{3}{2}$

•5 $\left(m_{tgt(P)\ x=3}\right) \times \left(m_{tgt(C)\ x=3}\right) = -1$

so "curves orthogonal"

•6 deal totally with other point

(*b*)

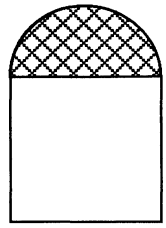
•7 $y - 7 = \frac{3}{2}(x+3)$

•8 circle centre $= \left(0, 11\frac{1}{2}\right)$

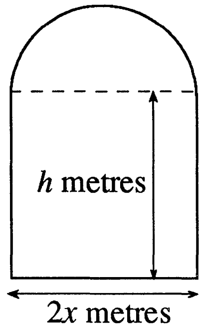
•9 $r^2 = \frac{117}{4}$

•10 $x^2 + \left(y - 11\frac{1}{2}\right)^2 = \frac{117}{4}$

A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.
The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	4	0.1					1	3	0.1		Source
(*b*)	5	1.3					2	3	1.3.15		**1996 Paper 2 Qu.11**

(*a*) •1 eg $2h + 2x + \text{semicircle} = 10$

•2 $h = \frac{1}{2}(10 - \pi x - 2x)$

•3 $L = 2 \times 2xh + \frac{1}{2}\pi x^2$

•4 $L = 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$

$L = 20x - 4x^2 - \frac{3}{2}\pi x^2$

(*b*) •5 $L' = 20 - 8x - 3\pi x$

•6 $L' = 0$

•7 $x = \dfrac{20}{3\pi + 8} = x_0 \ (= 1.148)$

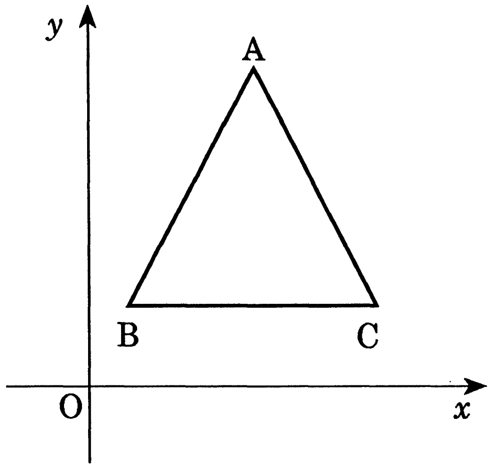
•8

x	x_0^-	x_0	x_0^+
L'	+	0	−

maximum at x_0

•9 $h = \dfrac{5\pi + 20}{3\pi + 8} \ (= 2.049)$

A triangle ABC has vertices A(4, 8), B(1, 2) and C(7, 2).



(*a*) Show that the triangle is isosceles. **(2)**

(*b*) (i) The altitudes AD and BE intersect at H, where D and E lie on BC and CA respectively. Find the coordinates of H. **(7)**

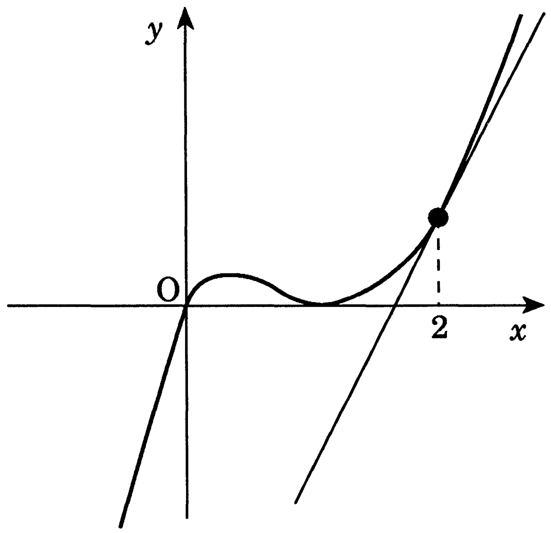
(ii) Hence show that H lies one quarter of the way up DA. **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.1
(*a*)	2	1.1					2		1.1.2		Source
(*b*)	8	1.1					8		1.1.10,	0.1	**1995 Paper 2 Qu.1**

(*a*)
- •[1] Calculate the length of the sides
- •[2] $AB = AC = \sqrt{3^2 + 6^2}$

(*b*)
- •[3] knows to find equ. of an altitude
- •[4] $m_{AC} = -2$
- •[5] $m_{BE} = \frac{1}{2}$
- •[6] $y - 2 = \frac{1}{2}(x - 1)$
- •[7] $x = 4$ stated or implied
- •[8] knows how to find intersection
- •[9] $H = \left(4, \frac{7}{2}\right)$
- •[10] $DA = 6$ and $DH = 1\frac{1}{2}$

The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(b) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(a)	4	1.3	4						1.3.9,	1.1.7	Source
(b)	5	2.1	5						2.1.2,	2.1.8	**1995 Paper 2 Qu.2**

(a)

•1 $\frac{dy}{dx} = \dots\dots\dots$

•2 $3x^2 - 4x + 1$

•3 $m_{x=2} = 5$

•4 $y - 2 = 5(x - 2)$

(b)

•5 equate 'y's

•6 $x^3 - 2x^2 - 4x + 8 = 0$

•7 e.g. synthetic division

•8 the appearance of:

$x^2 - 4$

or $x^2 - 4x + 4$

or ± 2

or $-2, 2, 2$

•9 $x = -2,\ y = -18$

Trees are sprayed weekly with the pesticide, KILLPEST, whose manufacturers claim it will destroy 65% of all pests. Between the weekly sprayings it is estimated that 500 new pests invade the trees.
A new pesticide, PESTKILL, comes onto the market. The manufacturers claim that it will destroy 85% of existing pests but it is estimated that 650 new pests per week will invade the trees.
Which pesticide will be more effective in the long term ? **(7)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
-	7	1.4			7				1.4.3,	1.4.4, 1.4.5	Source **1995 Paper 2 Qu.3**

(-) •1 $0 \cdot 35$ stated or implied

•2 $0 \cdot 35u_n + 500$

•3 $0 \cdot 15$ stated or implied

•4 $0 \cdot 15u_n + 650$

•5 $l = al + b$......... *or* limit $= \frac{b}{1-a}$.........

•6 limits $= 769$ and 765

•7 Limits are valid since $|a| < 1$ in both cases
and Pestkill is more effective