(*a*) (i) Diagram 1 shows part of the graph of the function f defined by $f(x) = b\sin ax^{\circ}$, where a and b are constants.
Write down the values of a and b.

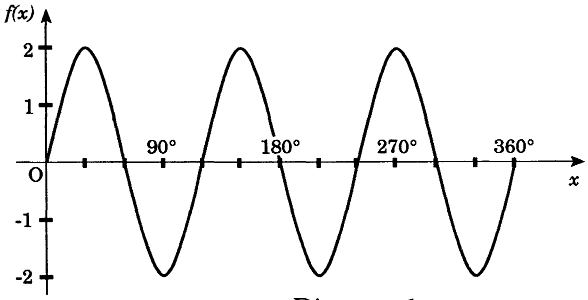


Diagram 1

(ii) Diagram 2 shows part of the graph of the function g defined by $g(x) = d\cos cx^{\circ}$, where c and d are constants.
Write down the values of c and d.

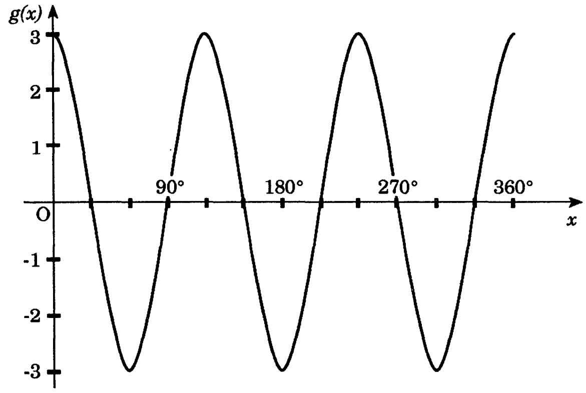


Diagram 2

(4)

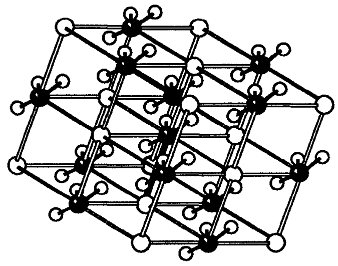
(*b*) The function h is defined by $h(x) = f(x) + g(x)$.
Show that $h(x)$ can be expressed in terms of a single trigonometric function of the form $q\sin(px + r)^{\circ}$ and find the values of p, q and r. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	**3.4**
(*a*)	4	2.3			4				2.3.2		Source **1995 Paper 2 Qu.4**
(*b*)	5	3.4			5				3.4.1		

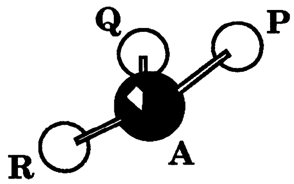
(*a*)
- •1 $a = 3$
- •2 $b = 2$
- •3 $c = 3$
- •4 $d = 3$

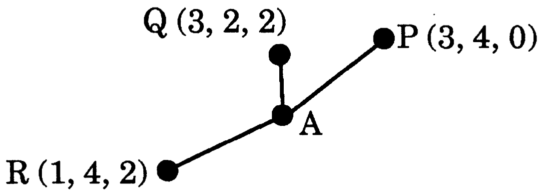
(*b*)
- •5 $p = 3$
- •6 $q\sin(px + r)^{\circ}$
  $= q\sin px^{\circ}\cos r^{\circ} + q\cos px^{\circ}\sin r^{\circ}$
- •7 $q = \sqrt{13}$
- •8 $q\cos r^{\circ} = 2,\ q\sin r^{\circ} = 3$
  or $\tan r^{\circ} = \frac{3}{2}$
- •9 $r = 56\cdot 3$

The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.





(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	4	3.1					4		3.1.11		Source
(*b*)	6	3.1					6		3.1.6,	3.1.3	**1995 Paper 2**
(*c*)	2	3.1					1	1	3.1.3,	0.1	**Qu.5**

(a)

•1 $PQ=\sqrt{8},\ RQ=\sqrt{8},$

•2 Use s.p.: $\overrightarrow{PQ}.\overrightarrow{RQ} = |\overrightarrow{PQ}|\,.\,|\overrightarrow{RQ}|\cos\theta$

•3 $\begin{pmatrix}0\\-2\\2\end{pmatrix}.\begin{pmatrix}2\\-2\\0\end{pmatrix}=4$

•4 60°

(b)

•5 $M=(2,3,2)$

•6 $\overrightarrow{PT}=\frac{2}{3}\overrightarrow{PM}$ or equivalent

•7 $\overrightarrow{PT}=\frac{2}{3}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ or equiv.

•8 $T=\left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$

•9 $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ stated or implied

•10 $PT=2\sqrt{\frac{2}{3}}, QT=2\sqrt{\frac{2}{3}}, RT=2\sqrt{\frac{2}{3}}$ or equivalent

(c)

•11 $PA=QA=RA=\sqrt{3}$

•12 A is in a different plane

A system of 3 equations in 3 unknowns can be solved by a method known as Gaussian Elimination as shown below.

Example

Solve the system of equations by Gaussian Elimination.

$$\begin{aligned} x + 2y - 3z &= 11 \\ 2x + 2y - z &= 11 \\ 3x - 2y + 4z &= -4 \end{aligned}$$

A Write out the coefficients in an array:

- Row 1
- Row 2
- Row 3

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 11 \\ 2 & 2 & -1 & 11 \\ 3 & -2 & 4 & -4 \end{array}\right)$$

B Keep Row 1 the same. Make Row 2 and Row 3 each begin with a zero by subtracting multiples of Row 1 from them.

- Row 1 is kept the same
- Row 2 becomes 'Row 2 – 2 x Row 1'
- Row 3 becomes 'Row 3 – 3 x Row 1'

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 11 \\ \mathbf{0} & \mathbf{-2} & \mathbf{5} & \mathbf{-11} \\ \mathbf{0} & \mathbf{-8} & \mathbf{13} & \mathbf{-37} \end{array}\right)$$

C Keep Row 1 and Row 2 the same. Make Row 3 begin with two zeros, by subtracting a multiple of Row 2 from it.

- Row 1 is kept the same
- Row 2 is kept the same
- Row 3 becomes 'Row 3 – 4 x Row 2'

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 11 \\ 0 & -2 & 5 & -11 \\ \mathbf{0} & \mathbf{0} & \mathbf{-7} & \mathbf{7} \end{array}\right) \begin{array}{l} \dots\dots\dots(1) \\ \dots\dots\dots(2) \\ \dots\dots\dots(3) \end{array}$$

D

- Line (3) gives $-7z = 7, \quad z = -1$
- Line (2) gives $-2y + 5z = -11$
  $-2y + (-5) = -11, \quad y = 3$
- Line (1) gives $x + 2y - 3z = 11$
  $x + 6 + 3 = 11, \quad x = 2$

So the solution is $x = 2, y = 3, z = -1$

Solve the following system of equations by Gaussian Elimination **as shown above.**

$$\begin{aligned} x - 2y + z &= 6 \\ 3x + y - z &= 7 \\ 4x - y + 2z &= 15 \end{aligned}$$

(7)

1995 Paper 2 Qu.6

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	4
-	7	0.1					7		0.1	Source **1995 Paper 2 Qu.6**

(-)

•[1]
$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 3 & 1 & -1 & 7 \\ 4 & -1 & 2 & 15 \end{array}\right)$$

•[2]
$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \end{array}\right)$$

•[3]
$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \\ 0 & 7 & -2 & -9 \end{array}\right)$$

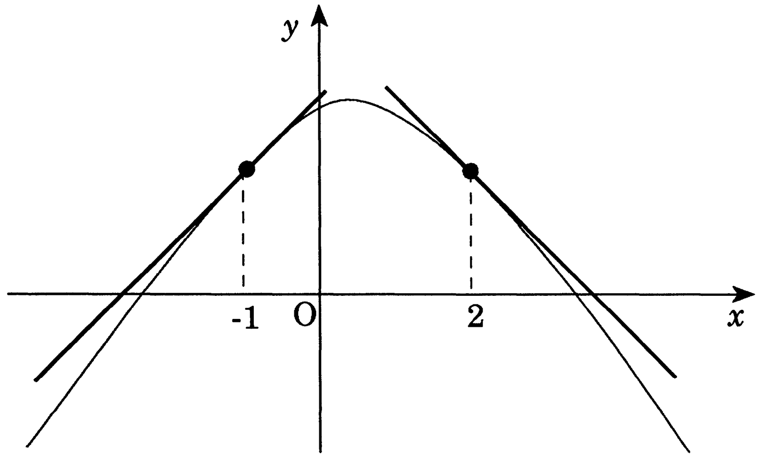
•[4]
$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \\ 0 & 0 & 2 & 2 \end{array}\right)$$

•[5] $2z = 2,\ z = 1$

•[6] $7y - 4z = -11,\ y = -1$

•[7] $x - 2y + z = 6,\ x = 3$

The parabola $y = ax^2 + bx + c$ crosses the y-axis at (0, 3) and has two tangents drawn, as shown in the diagram.

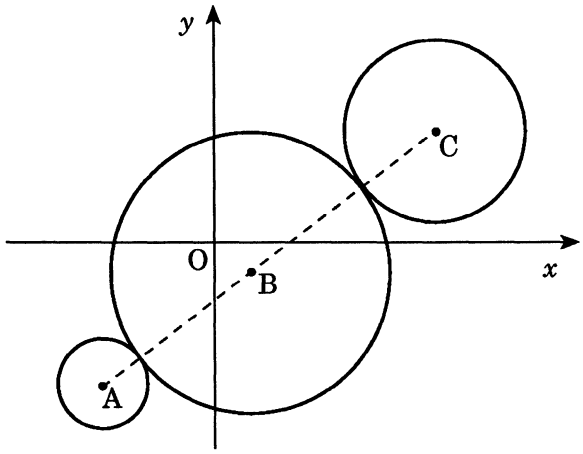


The tangent at $x = -1$ makes an angle of 45° with the positive direction of the x-axis and the tangent at $x = 2$ makes an angle of 135° with the positive direction of the x-axis.
Find the values of a, b and c. **(8)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
-	8	1.3	2	6					1.1.3,	1.3.7, 0.1	Source **1995 Paper 2 Qu.7**

(-)
- •[1] $c = 3$
- •[2] $2ax + b$
- •[3] $m = \tan 45° = 1$
- •[4] $-2a + b = 1$
- •[5] $m = \tan 135° = -1$
- •[6] $4a + b = -1$
- •[7] method for solving pr. of equ
- •[8] $a = -\frac{1}{3},\ b = \frac{1}{3}$

When newspapers were printed by lithograph, the newsprint had to run over three rollers, illustrated in the diagram by three circles. The centres A, B and C of the three circles are collinear.



The equations of the circumferences of the outer circles are

$(x+12)^2+(y+15)^2=25$ and $(x-24)^2+(y-12)^2=100$.

Find the equation of the central circle. **(8)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
-	8	3.1					8		2.4.1,	2.4.3, 3.1.6	Source **1995 Paper 2 Qu.8**

(-)

- •1 $(-12,-15)$ and $(24,12)$
- •2 radii are 5 and 10
- •3 AC = 45
- •4 radius = 15
- •5 B divides AC in ratio 4:5
- •6 $\vec{OB}=\frac{1}{9}\left[4\vec{OC}+5\vec{OA}\right]$ stated or implied
- •7 $\vec{OB}=\frac{1}{9}\left[4\begin{pmatrix}24\\12\end{pmatrix}+5\begin{pmatrix}-12\\-15\end{pmatrix}\right]$
- •8 $(x-4)^2+(y+3)^2=15^2$

(*a*) By writing $\sin 3x$ as $\sin(2x+x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. **(4)**

(*b*) Hence find $\int \sin^3 x \, dx$. **(4)**

part marks	Unit	non-calc		calc		calc neut		Content Reference :	3.2
		C	A/B	C	A/B	C	A/B	Main Additional	
(*a*) 4	2.3	2	2					2.3.2, 2.3.3	Source **1995 Paper 2 Qu.9**
(*b*) 4	3.2		4					3.2.4	

(*a*)

- •[1] $\sin 2x\cos x + \cos 2x \sin x$
- •[2] $2\sin x\cos x\cos x + \ldots\ldots\ldots$
- •[3] $\ldots\ldots\ldots + \left(1 - 2\sin^2 x\right)\sin x$
- •[4] $2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

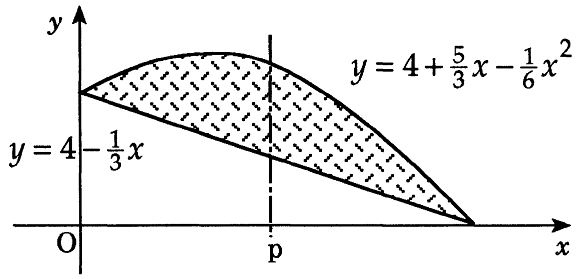
(*b*)

- •[5] $\int \frac{1}{4}(3\sin x - \sin 3x)\ dx$
- •[6] $-3\cos x$
- •[7] $+\cos 3x$
- •[8] $\div 3$

When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.

The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.



(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**

(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.
By using your answer to part (*a*), or otherwise, show that

$$p^3 - 18p^2 + 432 = 0.$$ **(3)**

(*c*) (i) Verify that $p = 6$ is a solution of this equation.
(ii) Find algebraically the other two solutions of this equation.
(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

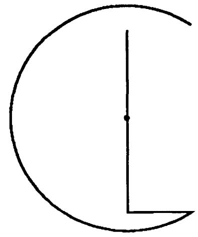
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	5	2.2	5						2.2.7		Source
(*b*)	3	2.2		3					2.2.5		**1995 Paper 2**
(*c*)	5	2.1	2	3					2.1.3		**Qu.10**

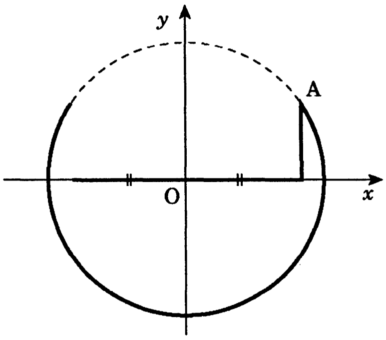
(a)
- •[1] Area under curve - area under line
- •[2] abscissae at intersection are 0 **and** 12
- •[3] $\int_0^{12} \left(4 + \frac{5}{3}x - \frac{1}{6}x^2 - \left(4 - \frac{1}{3}x\right)\right) dx$
- •[4] $\left[x^2 - \frac{1}{18}x^3\right]_0^{12}$ or equivalent
- •[5] 48

(b)
- •[6] $\int \ldots\ldots\ldots\ dx = 24$
- •[7] $\int_0^p \left(2x - \frac{1}{6}x^2\right) dx = 24$ or equivalent statement
- •[8] $p^2 - \frac{1}{18}p^3 = 24$

(c)
- •[9] "$f(6) = 0$" or equivalent
- •[10] divide by $(p-6)$
- •[11] $p^2 - 12p - 72$
- •[12] $p = 6 \pm \sqrt{108}$ or equivalent
- •[13] outside range 0 – 12

Linktown Church is considering designs for a logo for their Parish magazine. The 'C' is part of a circle and the centre of the circle is the mid-point of the vertical arm of the 'L'. Since the 'L' is clearly smaller than the 'C', the designer wishes to ensure that the total length of the arms of the 'L' is as long as possible.





The designer decides to call the point where the 'L' and 'C' meet A and chooses to draw co-ordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is $x^2 + y^2 = 20$.

(a) If A has co-ordinates (x,y), show that the total length T of the arms of the 'L' is given by $T = 2x + \sqrt{20 - x^2}$. **(1)**

(b) Show that for a stationary value of T, x satisfies the equation

$$x = 2\sqrt{20 - x^2}.$$ **(5)**

(c) By squaring both sides, solve this equation.
Hence find the greatest length of the arms of the 'L'. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.2
(a)	1	0.1						1	0.1		Source
(b)	5	3.2					1	4	3.2.2		**1995 Paper 2**
(c)	3	1.3					1	2	1.3.15		**Qu.11**

(a) •1 $T = x + x + y$ **and** $y^2 = 20 - x^2$

(b) •2 appearance of $\frac{dT}{dx} = 2 + \ldots\ldots$

•3 $\frac{1}{2}\left(20 - x^2\right)^{-\frac{1}{2}}$

•4 $\times -2x$

•5 $\frac{dT}{dx} = 0$ stated or implied

•6 completing proof

(c) •7 $x^2 = 4\left(20 - x^2\right)$

•8 $x = 4$ (accept $x = \pm 4$)

•9 justifying $x = 4$ gives $T_{max} = 10$

The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(*a*) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(*b*) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(*a*)	3	2.1	3						2.1.3		**1994 Paper 2 Qu.1**
(*b*)	4	2.1	4						2.1.3		

(*a*) •[1] strategy

eg

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & a \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

or $f(2) = 0 = 16 + 4 - 26 + a$

•[2] $a = 6$

•[3] $(0,6)$

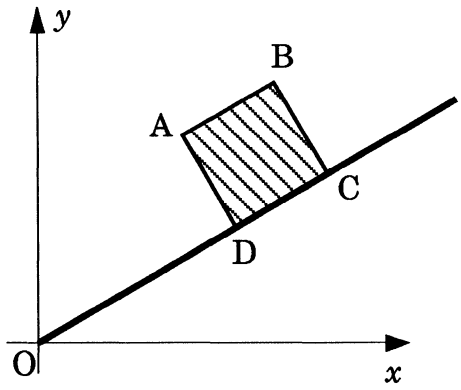
(*b*) •[4] $2x^2 + 5x - 3$

•[5] $(x+3)(2x-1)$

•[6] $x = -3,\ \frac{1}{2}$

•[7] $(-3,0),\ \left(\frac{1}{2},0\right)$

ABCD is a square. A is the point with coordinates (3,4) and ODC has equation $y = \frac{1}{2}x$.



(*a*) Find the equation of the line AD. **(3)**

(*b*) Find the coordinates of D. **(3)**

(*c*) Find the area of the square ABCD. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.1
(*a*)	3	1.1					3		1.1.9,	1.1.7	Source
(*b*)	3	0.1					3		0.1		**1994 Paper 2**
(*c*)	2	1.1					2		1.1.2		**Qu.2**

(*a*) •1 using $m_1 m_2 = -1$

•2 $m_{AD} = -2$

•3 $y - 4 = -2(x - 3)$

(*b*) •4 strategy for sim. equations

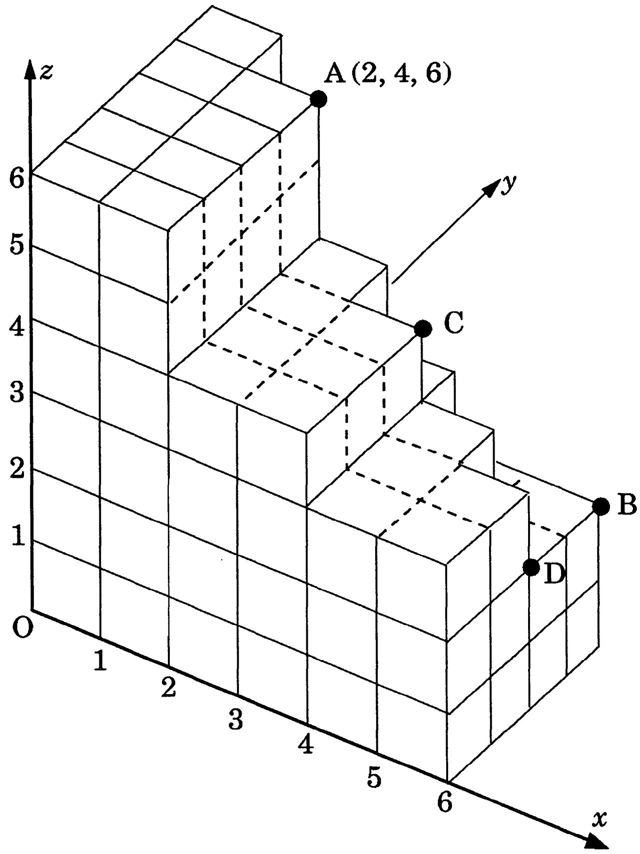
•5 $2x + y = 10$ or equiv

•6 (4,2)

(*c*) •7 strategy : find length of AD

•8 5

With coordinate axes as shown, the point A is (2,4,6).



(*a*) Write down the coordinates of B,C and D. **(3)**

(*b*) Show that C is the midpoint of AD. **(1)**

(*c*) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(*d*) Hence calculate the size of angle OAB. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	3	3.1			3				3.1.1		Source
(*b*)	1	3.1			1				3.1.6		**1994 Paper 2**
(*c*)	4	3.1			4				3.1.11		**Qu.3**
(*d*)	2	0.1			2				0.1		

(a) •1 One of B,C or D

•2 Remaining two of B, C and D

•3 B (6,4,2), C (4,3,4), D (6,2,2)

(b) •4 $\left(\frac{2+6}{2}, \frac{4+2}{2}, \frac{6+2}{2}\right)$

(c) •5 $\cos A\hat{O}B = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$ *or* $\frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$ or equivalents

•6 $\overrightarrow{OA}.\overrightarrow{OB} = 40$ *or* $AB^2 = 32$

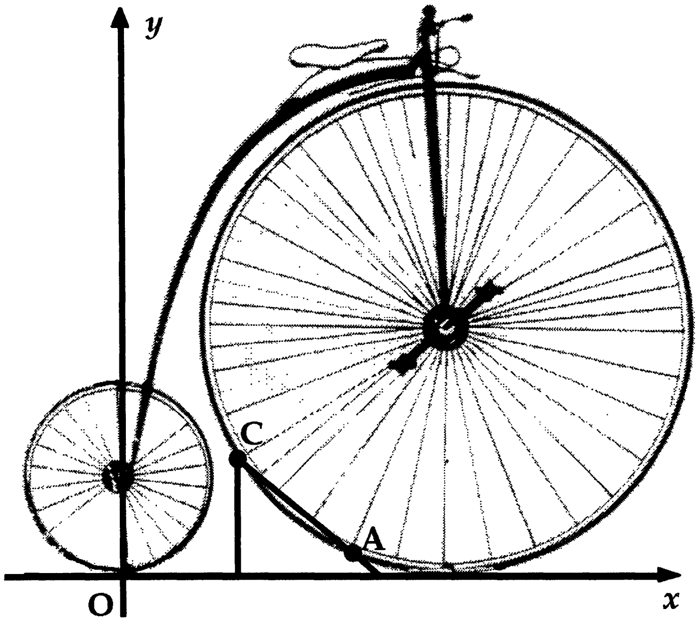
•7 $OA = \sqrt{56} = OB$

•8 $44°$

(d) •9 strategy: e.g. use isosceles Δ

•10 $68°$

A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.
With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is
$x^2 + y^2 - 6y = 0$ and
the equation of the front wheel is
$x^2 + y^2 - 28x - 20y + 196 = 0$.



(*a*) (i) Find the distance between the centres of the two wheels.

(ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. **(8)**

(*b*) B(7,3) is half-way between A and C, and P is the centre of the front wheel.

(i) Find the gradient of PB.

(ii) Hence find the equation of AC and the coordinates of A and C. **(8)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.4
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(*a*)	8	2.4			8				2.4.2,	1.1.2	**1994 Paper 2 Qu.4**
(*b*)	8	1.1			8				1.1.1,	1.1.9, 2.4.4	

(*a*)
- •1 centre (0, 3)
- •2 centre (14, 10)
- •3 distance between centres = $\sqrt{245}$
- •4 radius = 3
- •5 radius = 10
- •6 strategy (clearance = distance between centres minus sum of radii)
- •7 $\sqrt{245} - 13$
- •8 133 mm or equivalent

(*b*)
- •9 $m_{PB} = 1$
- •10 $m_{AC} = -1$
- •11 $y - 3 = -(x - 7)$ for AC
- •12 strategy: substitute
- •13 substituting correctly
- •14 *eg* $2x^2 - 28x + 96 = 0$
- •15 $x = 6,\ 8$ (*or* $y = 2, 4$)
- •16 $(6,4)$ **and** $(8,2)$

(*a*) Express $3\sin x^\circ - \cos x^\circ$ in the form $k\sin(x-\alpha)^\circ$, where $k > 0$ and $0 \le \alpha \le 90\cdot$ **(4)**

(*b*) Hence find algebraically the values of x between 0 and 180 for which $3\sin x^\circ - \cos x^\circ = \sqrt{5}$. **(4)**

(*c*) Find the range of values of x between 0 and 180 for which $3\sin x^\circ - \cos x^\circ \le \sqrt{5}$. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	3.4
(*a*)	4	3.4			4				3.4.1	Source
(*b*)	4	3.4			4				3.4.2	1994 Paper 2
(*c*)	2	3.4				2			3.4.2	Qu.5

(*a*)
- •1 $k(\sin x\cos\alpha - \cos x\sin\alpha)$ or equivalent
- •2 $k\cos\alpha = 3$ **and** $k\sin\alpha = 1$
- •3 $k = \sqrt{10}$
- •4 $\alpha = 18\cdot 4$

(*b*)
- •5 $\sqrt{10}\sin(x - 18\cdot 4)^\circ = \sqrt{5}$
- •6 $\sin(x - 18\cdot 4)^\circ = \frac{1}{\sqrt{2}}$ or equivalent
- •7 $63\cdot 4$
- •8 $153\cdot 4$

(*c*)
- •9 strategy stated or implied
- •10 $x \le 63.4$ **and** $x \ge 153.4$

EXAMPLE

(i) Let $f(x) = x^3 + 5x - 1$.
Since $f(0) = -1$ and $f(1) = 5$
the equation $f(x) = 0$ has a root in the interval $0 < x < 1$ because $f(0) < 0$ and $f(1) > 0$.

(ii) To find this root, the equation $x^3 + 5x - 1 = 0$ can be rearranged as follows :

$$x^3 + 5x - 1 = 0$$
$$x^3 + 5x = 1$$
$$x(x^2 + 5) = 1$$
$$x = \frac{1}{x^2 + 5}$$

We can write this result as a recurrence relation

$$x_{n+1} = \frac{1}{x_n^2 + 5}$$

and use it to find this root. In this example we will work to 3 decimal places and can therefore give the final answer to 2 decimal places.

(iii) For our first estimate, x_1, we use the mid-point of the interval $0 < x < 1$ [from part (i)].

$$x_1 = 0\cdot5, \quad x_2 = \frac{1}{0\cdot5^2 + 5} = 0\cdot190$$
$$x_2 = 0\cdot190 \quad x_3 = \frac{1}{0\cdot190^2 + 5} = 0\cdot199$$
$$x_3 = 0\cdot199 \quad x_4 = \frac{1}{0\cdot199^2 + 5} = 0\cdot198$$
$$x_4 = 0\cdot198 \quad x_5 = \frac{1}{0\cdot198^2 + 5} = 0\cdot198$$

Hence, correct to 2 decimal places, the root is $x = 0\cdot20$.

(*a*) Show that the equation $2x^3 + 3x - 1 = 0$ has a root in the interval $0 < x < 0\cdot5$. **(2)**

(*b*) By using the technique described above find this root correct to 2 decimal places. **(6)**

1994 Paper 2 Qu.6

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
											Source
(*a*)	2	0.1			2				0.1		**1994 Paper 2**
(*b*)	6	0.1			6				0.1		**Qu.6**

(*a*)

•[1] $f(0) = -1$ **and** $f(0 \cdot 5) = 0 \cdot 75$

•[2] "$f(0) < 0$ and $f(0.5) > 0$" or equiv. **explicitly stated**

(*b*)

•[3] $x = \frac{1}{2x^2+3}$

•[4] $x_1 = 0.25$

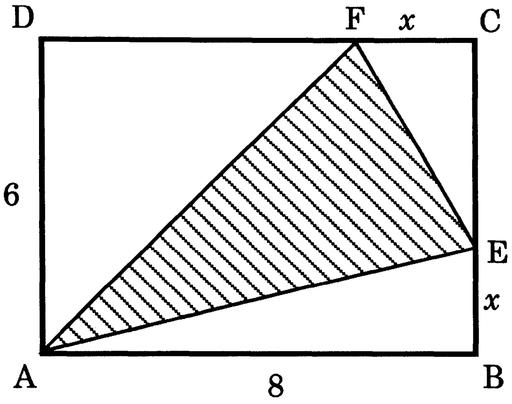
•[5] $x_2 = 0.32$

•[6] $x_3 = 0.312$ rounded to 3dp

•[7] $x_4 = 0.313$ *and* $x_5 = 0.313$

•[8] 0.31 correct to 2dp

An yacht club is designing its new flag. The flag is to consist of a red triangle on a yellow rectangular background.
In the yellow rectangle ABCD, AB measures 8 units and AD is 6 units. E and F lie on BC and CD, x units from B and C as shown in the diagram.



(*a*) Show that the area, H square units, of the red triangle AEF is given by $H(x) = 24 - 4x + \frac{1}{2}x^2$. **(4)**

(*b*) Hence find the greatest and least possible values of the area of triangle AEF. **(8)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.3
										Source
(*a*)	4	0.1	4						0.1	**1994 Paper 2**
(*b*)	8	1.3	3	5					1.3.15	**Qu.7**

(*a*)
- •[1] rectangle minus 3 triangles
- •[2] area of Δ's ADF and ABE
- •[3] area of Δ FCE
- •[4] 3 triangles : $24 + 4x - \frac{1}{2}x^2$ **or** $48 - 4x - 3x + \frac{1}{2}x^2 - 24 + 3x$

(*b*)
- •[5] $H'(x) = \ldots\ldots$
- •[6] $x - 4$
- •[7] put $H'(x) = 0$ **stated explicitly**
- •[8] $x = 4$ and $H = 16$
- •[9] justify minimum
- •[10] consider $x = 0$ **and** $x = 6$
- •[11] $H(0) = 24$, **and** $H(6) = 18$
- •[12] communication re greatest and least.

(*a*) $f(x) = 4x^2 - 3x + 5$.

Show that $f(x + 1)$ simplifies to $4x^2 + 5x + 6$ and find a similar expression for $f(x - 1)$.

Hence show that $\dfrac{f(x+1) - f(x-1)}{2}$ simplifies to $8x - 3$. **(5)**

(*b*) $g(x) = 2x^2 + 7x - 8$.

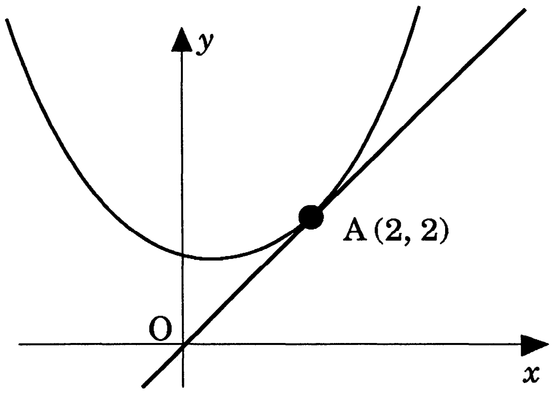
Find a similar expression for $\dfrac{g(x+1) - g(x-1)}{2}$. **(4)**

(*c*) By examining your answers for (*a*) and (*b*), **write down** the simplified expression for $\dfrac{h(x+1) - h(x-1)}{2}$, where $h(x) = 3x^2 + 5x - 1$. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
(*a*)	5	0.1					5		0.1		Source
(*b*)	4	0.1					4		0.1		**1994 Paper 2**
(*c*)	2	0.1					2		0.1		**Qu.8**

(*a*)	•1	$f(x+1) = 4(x+1)^2 - 3(x+1) + 5$
	•2	$4x^2 + 8x + 4 - 3x - 3 + 5$
	•3	$f(x-1) = 4(x-1)^2 - 3(x-1) + 5$
	•4	$f(x-1) = 4x^2 - 11x + 12$
	•5	$\frac{16x-6}{2}$
(*b*)	•6	$g(x+1) = 2(x+1)^2 + 7(x+1)$ *and* $g(x-1) = 2(x-1)^2 + 7(x-1) - 8$
	•7	$g(x+1) = 2x^2 + 11x + 1$
	•8	$g(x-1) = 2x^2 + 3x - 13$
	•9	$4x + 7$
(*c*)	•10	strategy stated or implied
	•11	$6x + 5$

(*a*) The point A(2, 2) lies on the parabola $y = x^2 + px + q$.
Find a relationship between p and q. **(1)**



(*b*) The tangent to the parabola at A is the line $y = x$. Find the value of p.
Hence find the equation of the parabola. **(6)**

(*c*) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value ? **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	1	0.1					1		0.1		Source
(*b*)	6	1.3					2	4	1.3.7,	0.1	**1994 Paper 2**
(*c*)	2	2.1						2	2.1.6		**Qu.9**

(*a*) •1 $2p + q = -2$

(*b*) •2 strategy

•3 $2x + p$

•4 gradient = 1, or equivalent

•5 $4 + p$

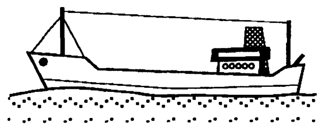
•6 $p = -3$

•7 $q = 4$

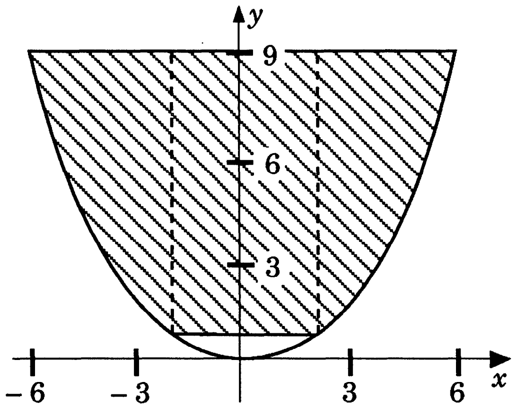
(*c*) •8 $\Delta = -7$

•9 $\sqrt{-7}$ means no roots

The cargo space of a small bulk carrier is 60m long.



The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y = \frac{1}{4}x^2$, $-6 \le x \le 6$, between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry.



(9)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
-	9	2.2					3	6	2.2.7,	0.1	**1994 Paper 2 Qu.10**

(-) •1 strategy: split into approp. parts

•2 $y = 1 \Rightarrow x = \pm 2$

•3 first rectangular area

•4 $9 - \frac{1}{4}x^2$ for integrand of shaped area

•5 $\int_2^5 \, dx$ for limits of shaped area

•6 for integrating...... $\left(9x - \frac{1}{12}x^3\right)$

•7 for evaluating...... $\left(\frac{56}{3}\right)$

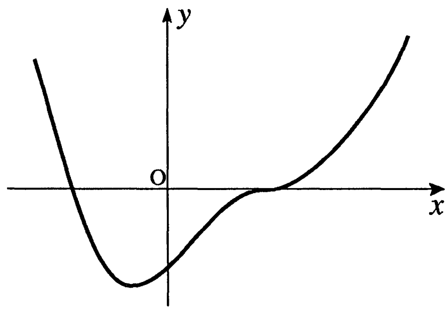
•8 total cross – sectional area $= \frac{208}{3} (m^2)$

•9 volume $= 4160 \ (m^3)$

The function f, whose incomplete graph is shown in the diagram, is defined by

$f(x) = x^4 - 2x^3 + 2x - 1$.

Find the coordinates of the stationary points and justify their nature. **(8)**



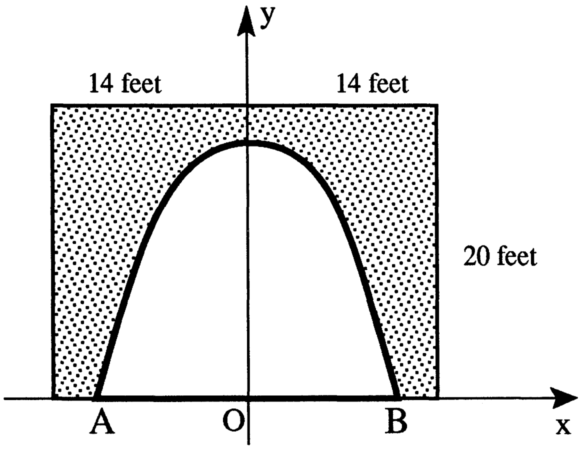
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
-	8	2.1					8		2.1.3,	1.3.12	Source **1993 Paper 2 Qu.1**

- •[1] for knowing to differentiate
- •[2] $f'(x) = 4x^3 - 6x^2 + 2$
- •[3] for putting $f'(x) = 0$
- •[4] for factorising **or** checking zeros
- •[5] $x = -\frac{1}{2},\ x = 1$
- •[6] $y = -\frac{27}{16},\ y = 0$
- •[7] completed nature table

x	$< -\frac{1}{2}$	$-\frac{1}{2}$	$> -\frac{1}{2}$	< 1	1	> 1
$f'(x)$	$-ve$	0	$+ve$	$+ve$	0	$+ve$
	╲	—	╱	╱	—	╱

- •[8] $(1,0)$ is pt. of inflexion, $\left(-\frac{1}{2}, -1\frac{11}{16}\right)$ is min t.p.

The concrete on the 20 feet by 28 feet rectangular facing of the entrance to an underground cavern is to be repainted.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 foot.The roof is in the form of a parabola with equation $y = 18 - \frac{1}{8}x^2$.

(*a*) Find the coordinates of the points A and B. **(2)**

(*b*) Calculate the total cost of repainting the facing at £3 per square foot. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
											Source
(*a*)	2	0.1	2						0.1		1993 Paper 2
(*b*)	4	2.2	4						2.2.6		Qu.2

(*a*) •1 $18 - \frac{1}{8}x^2 = 0$

•2 $x = \pm 12$

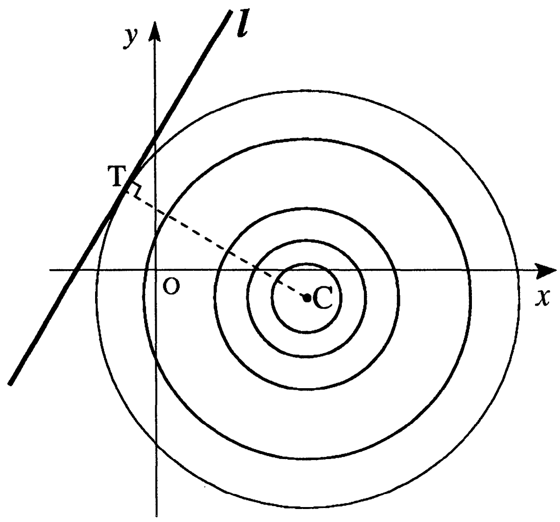
(*b*) •3 $Area = 2\int_0^{12} y \, dx$

•4 integrating

•5 288

•6 for knowing to subtract area of parabola from area of rectangle and multiply by 3.

In an experiment with a ripple tank, a series of concentric circles with centre C(4,–1) is formed as shown in the diagram.
The line l with equation $y = 2x + 6$ represents a barrier placed in the tank.
The largest complete circle touches the barrier at the point T.



(*a*) Find the equation of the radius CT. **(3)**

(*b*) Find the equation of the largest complete circle. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	3	1.1					3		1.1.9,	1.1.7	Source
(*b*)	5	2.4					5		2.4.3		1993 Paper 2 Qu.3

(*a*) •1 $m_l = 2$

•2 $m_r = -\frac{1}{2}$

•3 $y + 1 = -\frac{1}{2}(x - 4)$

(*b*) •4 $(x-4)^2 + (y+1)^2 = r^2$

•5 $(x-4)^2 + (2x+7)^2 = r^2$

•6 $5x^2 + 20x + (65 - r^2) = 0$

•7 $\Delta = 400 - 4 \times 5(65 - r^2) = 0$

•8 $r^2 = 45$

An array of numbers such as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is called a matrix. The eigenvalues of the matrix

$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are defined to be the roots of the equation $(a-x)(d-x)-bc=0$.

Example

> In order to find the eigenvalues of the matrix $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$
>
> solve $\quad (1-x)(2-x)-4x3=0$
>
> solution:
> $$2-3x+x^2-12=0$$
> $$x^2-3x-10=0$$
> $$(x+2)(x-5)=0$$
> $$x=-2 \text{ or } x=5$$
>
> so the eigenvalues of **B** are –2 and 5

(*a*) Find the eigenvalues of $\mathbf{C} = \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$. **(3)**

(*b*) Find the value of *t* for which the eigenvalues of the matrix $\mathbf{D} = \begin{pmatrix} 3 & -1 \\ t & 1 \end{pmatrix}$ are equal. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	0.1
(*a*)	3	0.1					3		0.1		Source
(*b*)	5	2.1					5		2.1.7,	0.1	**1993 Paper 2 Qu.4**

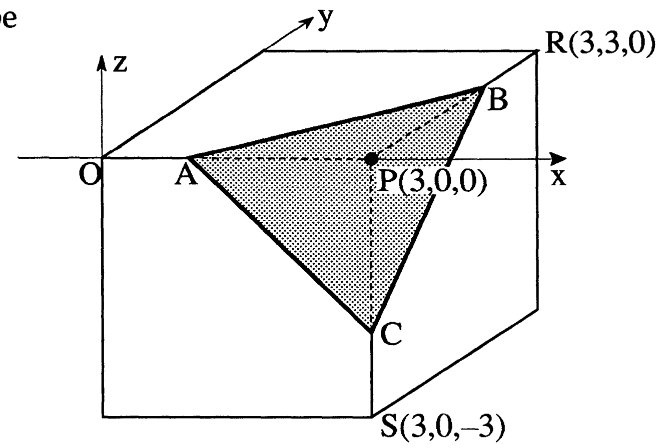
(*a*)

- •[1] $(3-x)(5-x)-2\times 4=0$
- •[2] $x^2-8x+7=0$
- •[3] eigenvalues are 1, 7

(*b*)

- •[4] $(3-x)(1-x)+t=0$
- •[5] $x^2-4x+(3+t)=0$
- •[6] $\Delta=0$ for equal roots or equiv.
- •[7] $\Delta=16-4\times 1\times(3+t)$ or equiv.
- •[8] $t=1$

A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC. Coordinate axes have been introduced as shown in the diagram.
The point A divides OP in the ratio 1:2.
Points B and C similarly divide RP and SP respectively in the ratio 1:2.



(*a*) Find the coordinates of A, B and C. **(3)**

(*b*) Calculate the area of triangle ABC. **(4)**

(*c*) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

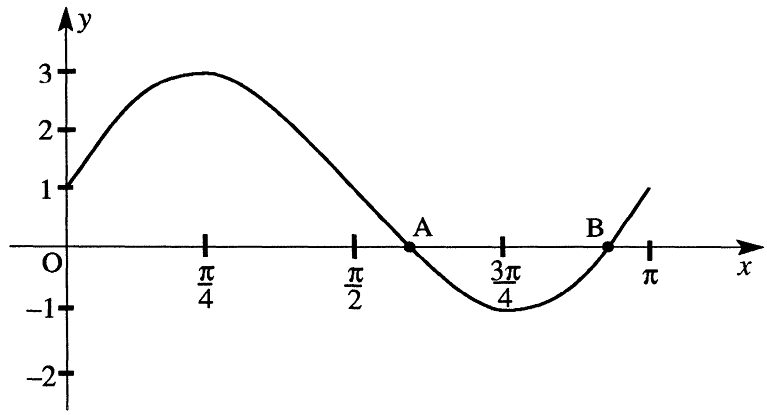
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	3.1
(*a*)	3	3.1			3				3.1.6		Source
(*b*)	4	3.1			4				3.1.3,	0.1	**1993 Paper 2**
(*c*)	5	0.1			5				0.1		**Qu.5**

(*a*)
- •[1] $A(1,0,0)$
- •[2] $B(3,2,0)$
- •[3] $C(3,0,-2)$

(*b*)
- •[4] strategy for area of triangle and attempt to calculate parts
- •[5] 60° or altitude $= \sqrt{6}$
- •[6] side $= 2\sqrt{2}$
- •[7] using chosen formula correctly

(*c*)
- •[8] 54 unit^2 for cube
- •[9] know how to calculate s.a of crystal
- •[10] area of 1 pentagonal face = 7 unit^2
- •[11] 51.5 unit^2 for crystal $(48 + 2\sqrt{3})$
- •[12] strategy for finding % decrease

The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.



(*a*) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. **(5)**

(*b*) The points (0, 2) and (π, 0) are joined by a straight line l. In how many points does l intersect the given graph ? **(1)**

(*c*) C is the point on the given graph with an x-coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line l. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(*a*)	5	2.3	3	2					2.3.1		Source
(*b*)	1	0.1	1						0.1		1993 Paper 2
(*c*)	3	0.1		3					0.1		Qu.6

(*a*) •[1] $2\sin 2x + 1 = 0$

•[2] $\sin 2x = -\frac{1}{2}$

•[3] for any valid sol of equ. in form $\sin ax = -\frac{b}{c}$

•[4] $\left(\frac{7\pi}{12}, 0\right)$

•[5] $\left(\frac{11\pi}{12}, 0\right)$

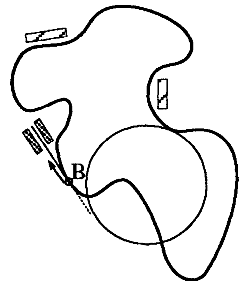
(*b*) •[6] 3

(*c*) •[7] $y_C = 1$

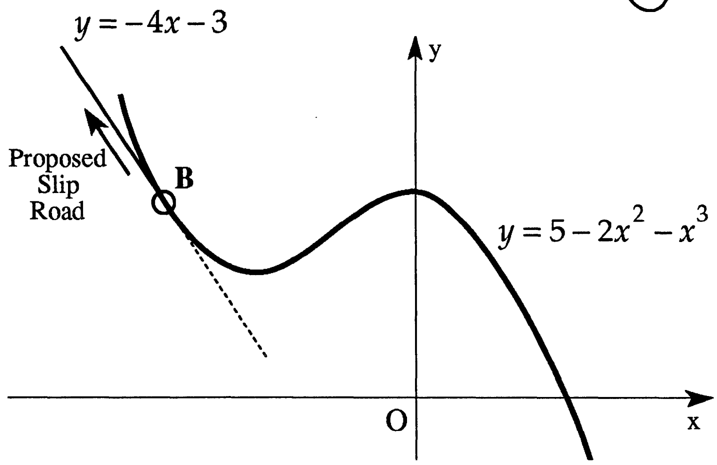
•[8] for a strategy to make a decision about C

•[9] for making a consistent decision about C

The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.



Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	7	2.1	7						2.1.8,	2.1.3	Source **1993 Paper 2 Qu.7**
(*b*)	1	2.1		1					2.1.8		

(*a*) •1 equating expressions for y

•2 re-arranging cubic...... "..."= 0

•3 strategy for solving cubic

•4 first linear factor

•5 quadratic factor

•6 $x = -2,\ 2$

•7 intersection at (– 2, 5)

(*b*) •8 double root $\Rightarrow$ tangency **or** $y'(-2) = -4$ = gradient of line

Secret Agent 004 has been captured and his captors are giving him a 25 milligram dose of a truth serum every 4 hours. 15% of the truth serum present in his body is lost every hour.

(*a*) Calculate how many milligrams of serum remain in his body after 4 hours (that is immediately **before** the second dose is given). **(3)**

(*b*) It is known that the level of serum in the body has to be continuously above 20 milligrams before the victim starts to confess. Find how many doses are needed before the captors should begin their interrogation. **(3)**

(*c*) Let u_n be the amount of serum (in milligrams) in his body just **after** his n^{th} dose. Show that $u_{n+1} = 0\cdot522u_n + 25$. **(1)**

(*d*) It is also known that 55 milligrams of this serum in the body will prove fatal, and the captors wish to keep Agent 004 alive. Is there any maximum length of time for which they can continue to administer this serum and still keep him alive ? **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.4
(*a*)	3	1.4			3				1.4.1		Source
(*b*)	3	1.4			3				1.4.1		**1993 Paper 2**
(*c*)	1	1.4			1				1.4.3		**Qu.8**
(*d*)	4	1.4			3	1			1.4.4,	1.4.5	

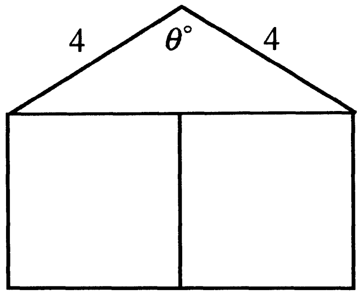
(*a*)
- •1 strategy for each hour (e.g. using 0·85)
- •2 using strategy 4 times (e.g. ($0\cdot85^4$)
- •3 13·05

(*b*)
- •4 apply a correct dose strategy
- •5 a relevant sequence e.g. 13·05, 19·86, 23·4,
  or 25, 38·05, 44·9, 48·4
- •6 3 doses

(*c*)
- •7 valid explanation i.e. $(0\cdot85)^4 = 0\cdot522$ explicitly stated

(*d*)
- •8 statement that limit exists because $(0\cdot85)^4 < 1$
- •9 $\therefore l = 0\cdot522l + 25$ or using $l = \frac{b}{1-a}$
- •10 l = 52·3
- •11 52·3 < 55 so no maximum length of time

A builder has obtained a large supply of 4 metre rafters. He wishes to use them to build some holiday chalets. The planning department insists that the gable end of each chalet should be in the form of an isosceles triangle surmounting two squares, as shown in the diagram.



(*a*) If θ ° is the angle shown in the diagram and A is the area (in square metres) of the gable end , show that $A = 8\left(2 + \sin\theta^\circ - 2\cos\theta^\circ\right)$. **(5)**

(*b*) Express $8\sin\theta^\circ - 16\cos\theta^\circ$ in the form $k\sin(\theta - \alpha)^\circ$. **(4)**

(*c*) Find algebraically the value of θ for which the area of the gable end is 30 square metres. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.4
(*a*)	5	0.1			1	4			0.1,	2.3.3	Source
(*b*)	4	3.4			4				3.4.1		**1993 Paper 2**
(*c*)	4	3.4			1	3			3.4.2		**Qu.9**

(*a*)
- •[1] area of triangle = $\frac{1}{2} \times 4 \times 4\sin\theta$ *or* $2 \times \frac{1}{2} \times 4\sin\frac{\theta}{2} \times 4\cos\frac{\theta}{2}$
- •[2] strategy for finding length of side of square or rectangle
- •[3] for length of side or (length of side)^2 of square/rectangle
- •[4] area of rectangle
- •[5] simplifying

(*b*)
- •[6] strategy including expansion of $k\sin(\theta - \alpha)$
- •[7] $k\cos\alpha = 8$ & $k\sin\alpha = 16$
- •[8] $k = 8\sqrt{5}$ or equiv.
- •[9] $\tan\alpha = 2 \Rightarrow \alpha = 63\cdot4$

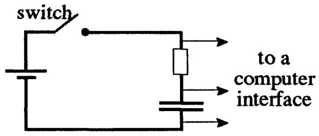
(*c*)
- •[10] $8(2 + \sin\theta - 2\cos\theta) = 30$
- •[11] $8\sqrt{5}\sin(\theta - 63\cdot4)^\circ = 14$
- •[12] $\sin(\theta - 63\cdot4)^\circ = 0\cdot783$
- •[13] $\theta = 51\cdot5 + 63\cdot4 = 114\cdot9$

Note : For •[3] various forms of the length are

square: $4\sin\frac{\theta}{2}, \frac{2\sin\theta}{\sin\left(90 - \frac{\theta}{2}\right)}, \sqrt{16 - 16\cos^2\frac{\theta}{2}}$

rect: $\frac{4\sin\theta}{\sin\left(90 - \frac{\theta}{2}\right)}, \sqrt{32 - 32\cos\theta}$

When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds).This graph is shown in fig. 1.



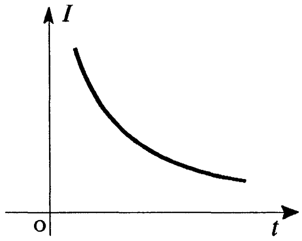


figure 1

In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.

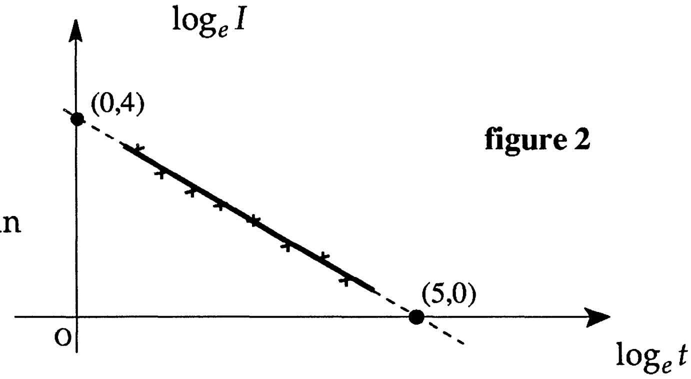


figure 2

(*a*) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. **(3)**

(*b*) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	3	1.1			3				1.1.1,	1.1.7	Source
(*b*)	4	3.3				4			3.3.6		**1993 Paper 2 Qu.10**

(*a*) •1 $m = -\frac{4}{5}$ stated or implied

•2 $y = mx + 4$ stated or implied

•3 $\log_e I = -\frac{4}{5}\log_e t + 4$

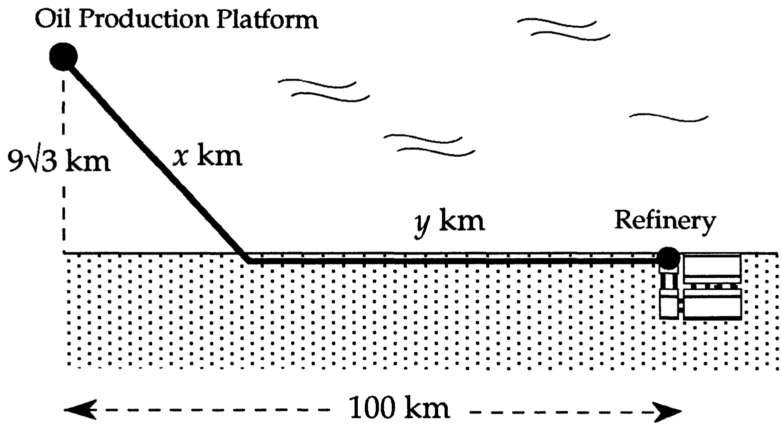
(*b*) •4 $\log_e t^{-\frac{4}{5}}$

•5 $\log_e 54.6$

•6 $\log_e 54.6t^{-\frac{4}{5}}$

•7 $I = 54.6t^{-0.8}$

An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(*a*) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$ **(3)**

(*b*) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.2
(*a*)	3	0.1	1	2					0.1		Source
(*b*)	7	1.3	1	6					1.3.15,	3.2.2	**1993 Paper 2 Qu.11**

(*a*) •1 $C = 2x + y$

•2 $\sqrt{x^2 - (9\sqrt{3})^2}$

•3 for completing proof

(*b*) •4 knowing to differentiate

•5 $\frac{1}{2}\left(x^2 - 243\right)^{-\frac{1}{2}}$

•6 $\times\ 2x$

•7 $C'(18) = 0$

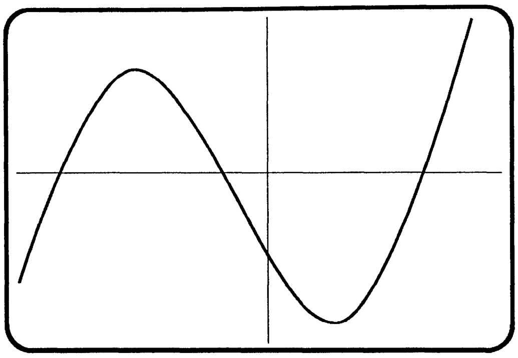
•8 justification of minimum e.g. nature table

•9 $C = 127$

•10 $x + y = 109$

	18^-	18	18^+
$C'(x)$	–	0	+
	\	—	/
		minimum	

The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**

(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(*c*) Find the turning points and justify their nature. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	**2.1**
(*a*)	3	2.1	3						2.1.3		Source
(*b*)	2	1.2	2						1.2.9		**1992 Paper 2**
(*c*)	6	1.3	6						1.3.12		**Qu.1**

(*a*) •1 any linear factor

•2 corresponding quadratic factor

•3 $f(x) = (x+1)(x-4)(x+4)$

(*b*) •4 For all 3 points on x-axis

•5 (0,–16)

(*c*) •6 $f'(x) = 3x^2 + 2x - 16$

•7 use $f'(x) = 0$

•8 $x = 2$, **and** $x = -\frac{8}{3}$

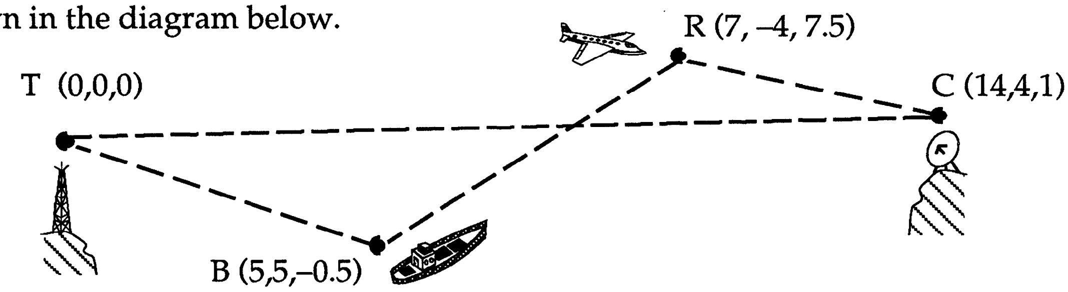
•9 $y = -36$, **and** $y = \frac{400}{27}(14.8)$

•10

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	−	−	0	+
	⋰	⋯	⋱	⋱	⋯	⋰

•11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2,-36)$

Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	3	3.1			3				3.1.3		Source
(*b*)	2	3.1			2				3.1.3		**1992 Paper 2**
(*c*)	3	3.1			3				3.1.10		**Qu.2**
(d)	5	3.1			5				3.1.11		

(*a*) •1 Strategy: use vectors or 3-D distance formula

•2 $\overrightarrow{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ *or* $BR^2 = 2^2 + 7^2 + 4^2$

•3 answer

(*b*) •4 $|\overrightarrow{MR}| = \sqrt{115.25}$ or equivalent

•5 answer

(*c*) •6 know to use a scalar product

•7 $\overrightarrow{TC}.\overrightarrow{BR} = 0$

•8 communication: 0 ⇔ perpendicularity

(*d*) •9 Strategy: know to use

$\cos T\hat{C}R = \dfrac{\overrightarrow{TC}.\overrightarrow{RC}}{|TC||RC|}$ or equiv.

•10 $\overrightarrow{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ and $\overrightarrow{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

•11 $\sqrt{161}$ and $\sqrt{65}$

•12 $\overrightarrow{TC}.\overrightarrow{RC} = 82$

•13 36.7°

Biologists calculate that when the concentration of a particular chemical in a sea loch reaches 5 milligrams per litre (mg/l) the level of pollution endangers the life of the fish.
A factory wishes to release waste containing this chemical into the loch. It is claimed that the discharge will not endanger the fish.
The Local Authority is supplied with the following information:

1. The loch contains none of this chemical at present.
2. The factory manager has applied to discharge effluent once per week which will result in an increase in concentration of 2.5 mg/l of the chemical in the loch.
3. The natural tidal action will remove 40% of the chemical from the loch every week.

(*a*) Show that this level of discharge would result in fish being endangered. **(3)**

When this result is announced, the company agrees to install a cleaning process that reduces the concentration of chemical released into the loch by 30%.

(*b*) Show the calculations you would use to check this revised application. Should the Local Authority grant permission ? **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.4
(*a*)	3	1.4			3				1.4.3		Source
(*b*)	5	1.4			5				1.4.3,	1.4.5	**1992 Paper 2 Qu.3**

(*a*) •1 0.6 stated/implied
•2 $u_{n+1} = 0.6u_n + 2.5$
•3 communication: ie 6.25 ⇒ danger

(*b*) •4 0.7 x 2.5 = 1.75
•5 2.8, 3.43, 3.808
•6 $u_{n+1} = 0.6u_n + 1.75$
•7 limit = 4.375
•8 communication: ie 4.375 ⇒ allow/disallow

(*a*) For a particular radioactive substance the mass m (in grams) at time t (in years) is given by

$$m = m_0 e^{-0.02t}$$

where m_0 is the original mass.

If the original mass is 500 grams, find the mass after 10 years. **(2)**

(*b*) The half-life of any material is the time taken for half of the mass to decay.
Find the half-life of this substance. **(3)**

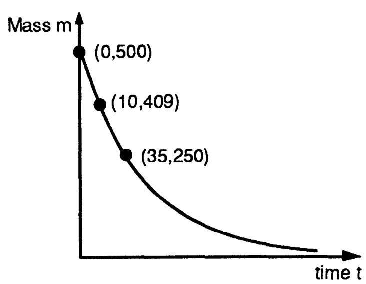
(*c*) Illustrate **ALL** of the above information on a graph. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.3
(*a*)	2	3.3			2				3.3.4		Source
(*b*)	3	3.3			1	2			3.3.4		**1992 Paper 2**
(*c*)	3	1.2			1	2			1.2.5		**Qu.4**

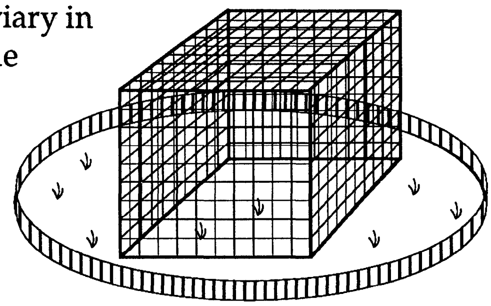
(*a*) •1 $m = 500e^{-0.02\times10}$
•2 409.37 grams

(*b*) •3 $250 = 500e^{-0.02t}$
•4 $\ln 250 = \ln 500 - 0.02t \times 1$ or equiv.
•5 34.7 years

(*c*) •6 any **two** of the 3 points
•7 the remaining point
•8 a decreasing curve



The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3.



(*a*) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2000}{x}.$$

(4)

(*b*) Find the dimensions of the aviary to ensure that the cost of netting is minimised. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	4	0.1	2	2					0.1		Source
(*b*)	6	1.3	4	2					1.3.15		**1992 Paper 2 Qu.5**

(*a*)
- •[1] introduce height specific to this cuboid
- •[2] $h = \frac{500}{x^2}$
- •[3] $A = x^2 + 4xh$
- •[4] $A = x^2 + 4x.\frac{500}{x^2}$ explicitly stated

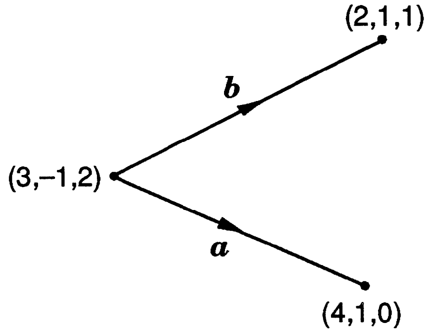
(*b*)
- •[5] $A'(x) = \ldots\ldots$
- •[6] $2x - 2000x^{-2}$
- •[7] $A'(x) = 0$ specifically stated
- •[8] $x = 10$
- •[9] justify minimum e.g. with table
- •[10] dimensions of 10 by 10 by 5

The *vector product*, $\boldsymbol{a} \times \boldsymbol{b}$, of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined by

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

EXAMPLE

when $\boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ then $\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} 2\times2-3\times0 \\ 3\times(-1)-1\times2 \\ 1\times0-2\times(-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix}$



(*a*) If $\boldsymbol{a}$ and $\boldsymbol{b}$ are as shown in the diagram and $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$, evaluate $\boldsymbol{c}$. **(3)**

(*b*) By considering $\boldsymbol{a}.\boldsymbol{c}$ and $\boldsymbol{b}.\boldsymbol{c}$, what can be concluded about $\boldsymbol{c}$? **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	3	0.1					3		0.1		Source **1992 Paper 2 Qu.6**
(*b*)	4	3.1					4		3.1.9, 3.1.10		

(*a*) •1 $a = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

•2 substitute in the rule for a x b

•3 answer

(*b*) •4 evaluate $\underline{a}.\underline{c}$

•5 evaluate $\underline{b}.\underline{c}$

•6 a statement that $\underline{a}$ is perpendicular to $\underline{c}$

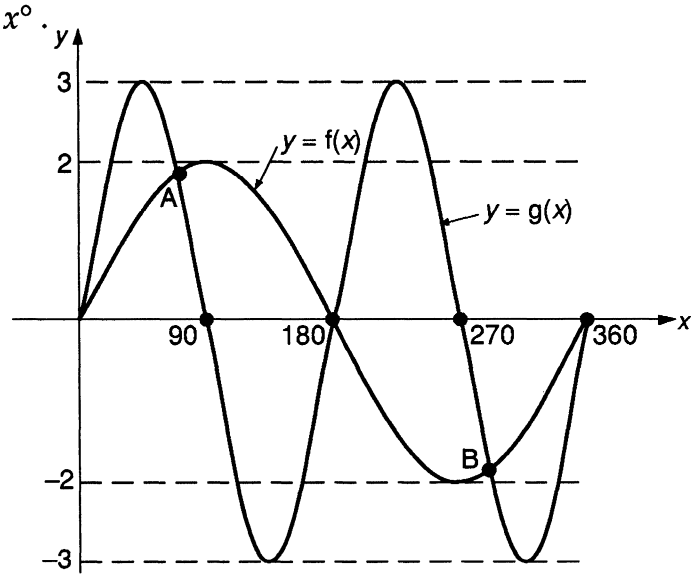
•7 a statement that $\underline{b}$ is perpendicular to $\underline{c}$

(*a*) Solve the equation $3\sin 2x° = 2\sin x°$ for $0 \le x \le 360$ **(4)**

(*b*) The diagram below shows parts of the graphs of sine functions *f* and *g*. State expressions for $f(x)$ and $g(x)$. **(1)**

(*c*) Use your answers to part (*a*) to find the co-ordinates of A and B. **(2)**

(*d*) Hence state the values of *x* in the interval $0 \le x \le 360$ for which $3\sin 2x° < 2\sin x°$. **(3)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.3
(*a*)	4	2.3			4				2.3.5	Source
(*b*)	1	1.2			1				1.2.7	**1992 Paper 2**
(*c*)	2	1.2			2				1.2.9	**Qu.7**
(*d*)	3	1.2			2	1			1.2.10	

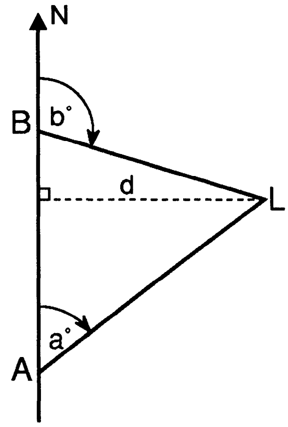
(*a*)
- •1 strategy: ie $\sin 2x = 2\sin x\cos x$
- •2 $\sin x = 0$ AND $\cos x = \frac{1}{3}$
- •3 0, 180 AND 360
- •4 70.5 AND 289.5 AND no other angles

(*b*)
- •5 $f(x) = 2\sin x°,\ g(x) = 3\sin 2x°$

(*c*)
- •6 $x = 70.5$ AND 289.5
- •7 $y = 1.89$ AND -1.89

(*d*)
- •8 70.5 AND 180
- •9 289.5 AND 360
- •10 use inequality signs logically to connect the points of intersection (ie **not** for $180 < x < 70.5$)

A ship is sailing due north at a constant speed. When at position A, lighthouse L is observed on a bearing of $a°$. One hour later, when the ship is at position B, the lighthouse is on a bearing of $b°$. The shortest distance between the ship and the lighthouse during this hour was d miles.



(*a*) Prove that $AB = \frac{d}{\tan a°} - \frac{d}{\tan b°}$. **(2)**

(*b*) Hence prove that $AB = \frac{d\sin(b-a)°}{\sin a° \sin b°}$. **(3)**

(*c*) Calculate the shortest distance from the ship to the lighthouse when the bearings $a°$ and $b°$ are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(*a*)	2	0.1			1	1			0.1		Source
(*b*)	3	2.3				3			2.3.4		**1992 Paper 2**
(*c*)	3	0.1			3				0.1		**Qu.8**

(*a*) •[1] $CA = \frac{d}{\tan a}$

•[2] $CB = \frac{d}{\tan(180-b)}$

(*b*) •[3] $AB = \frac{d}{\frac{\sin a}{\cos a}} - \frac{d}{\frac{\sin b}{\cos b}}$

•[4] $\frac{d\cos a}{\sin a} - \frac{d\cos b}{\sin b}$

•[5] $\frac{d\sin b\cos a - d\cos b\sin a}{\sin a\sin b}$

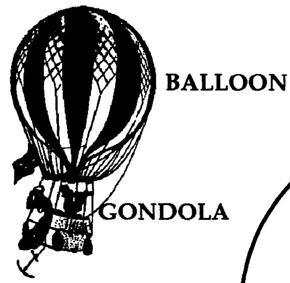
(*c*) •[6] AB = 14

•[7] 1.577 or 0.634

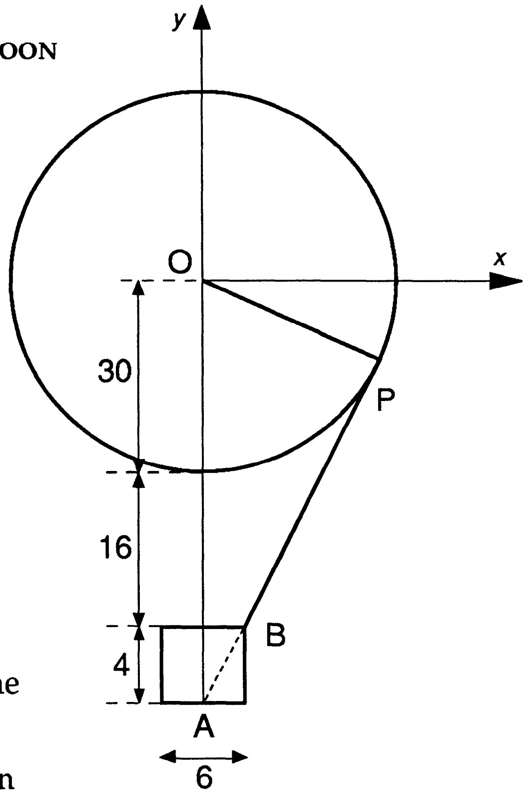
(comes from AB = 1.577d or d = 0.634 AB)

•[8] 8.9 miles

A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.



Co-ordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.



(*a*) Find the equation of the cable PB. **(3)**

(*b*) State the equation of the circle representing the balloon. **(1)**

(*c*) Prove that this cable is a tangent to the balloon and find the co-ordinates of the point P. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.4
(*a*)	3	1.1					3		1.1.1,	1.1.7	Source
(*b*)	1	2.4					1		2.4.3		**1992 Paper 2**
(*c*)	5	2.4					2	3	2.4.4		**Qu.9**

(*a*)
- •[1] Strategy: know to find m
- •[2] $m = \frac{4}{3}$
- •[3] $y + 46 = \frac{4}{3}(x - 3)$

(*b*)
- •[4] $x^2 + y^2 = 900$ or equivalent

(*c*)
- •[5] Strategy: know to substitute
- •[6] $x^2 + \left(\frac{4}{3}x - 50\right)^2 = 900$
- •[7] $(x - 24)^2$ or evaluate the discriminant
- •[8] communication relating to tangency
- •[9] $(24, -18)$