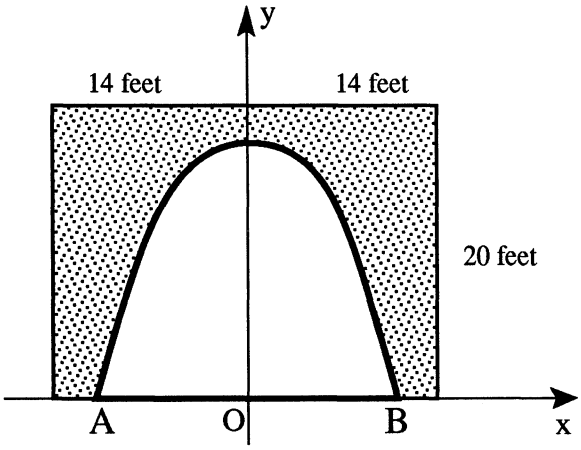
The concrete on the 20 feet by 28 feet rectangular facing of the entrance to an underground cavern is to be repainted.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 foot.The roof is in the form of a parabola with equation $y = 18 - \frac{1}{8}x^2$.

(*a*) Find the coordinates of the points A and B. **(2)**

(*b*) Calculate the total cost of repainting the facing at £3 per square foot. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	2	0.1	2						0.1		Source
(*b*)	4	2.2	4						2.2.6		**1993 Paper 2 Qu.2**

(*a*) •[1] $18 - \frac{1}{8}x^2 = 0$

•[2] $x = \pm 12$

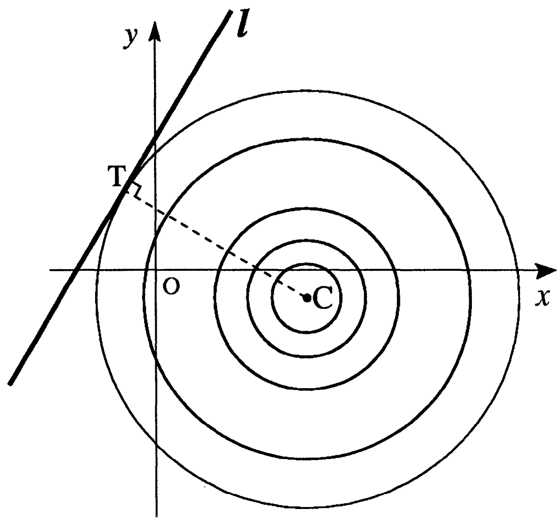
(*b*) •[3] $Area = 2\int_0^{12} y \, dx$

•[4] integrating

•[5] 288

•[6] for knowing to subtract area of parabola from area of rectangle and multiply by 3.

In an experiment with a ripple tank, a series of concentric circles with centre C(4,–1) is formed as shown in the diagram.
The line l with equation $y = 2x + 6$ represents a barrier placed in the tank.
The largest complete circle touches the barrier at the point T.



(*a*) Find the equation of the radius CT. **(3)**

(*b*) Find the equation of the largest complete circle. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	3	1.1					3		1.1.9,	1.1.7	Source
(*b*)	5	2.4					5		2.4.3		**1993 Paper 2 Qu.3**

(*a*)
- •[1] $m_l = 2$
- •[2] $m_r = -\frac{1}{2}$
- •[3] $y + 1 = -\frac{1}{2}(x - 4)$

(*b*)
- •[4] $(x-4)^2 + (y+1)^2 = r^2$
- •[5] $(x-4)^2 + (2x+7)^2 = r^2$
- •[6] $5x^2 + 20x + (65 - r^2) = 0$
- •[7] $\Delta = 400 - 4 \times 5(65 - r^2) = 0$
- •[8] $r^2 = 45$

An array of numbers such as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is called a matrix. The eigenvalues of the matrix

$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are defined to be the roots of the equation $(a-x)(d-x)-bc=0$.

EXAMPLE

> In order to find the eigenvalues of the matrix $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$
>
> solve $(1-x)(2-x)-4x3=0$
>
> solution:
> $$2-3x+x^2-12=0$$
> $$x^2-3x-10=0$$
> $$(x+2)(x-5)=0$$
> $$x=-2 \text{ or } x=5$$
>
> so the eigenvalues of **B** are –2 and 5

(*a*) Find the eigenvalues of $\mathbf{C} = \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$. **(3)**

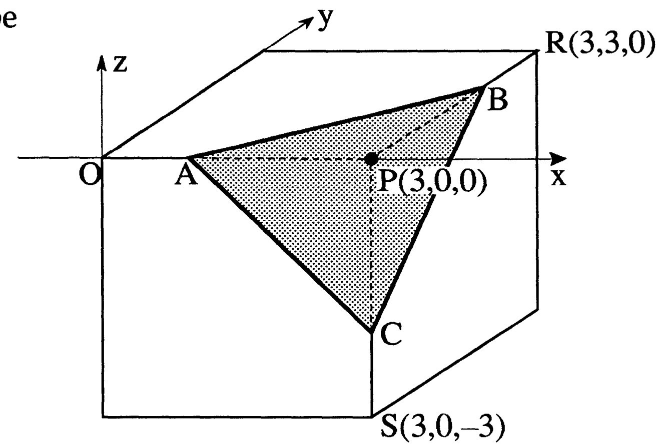
(*b*) Find the value of *t* for which the eigenvalues of the matrix $\mathbf{D} = \begin{pmatrix} 3 & -1 \\ t & 1 \end{pmatrix}$ are equal. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	0.1
(*a*)	3	0.1					3		0.1		Source
(*b*)	5	2.1					5		2.1.7,	0.1	**1993 Paper 2 Qu.4**

(*a*)
- •[1] $(3-x)(5-x)-2\times4=0$
- •[2] $x^2-8x+7=0$
- •[3] eigenvalues are 1, 7

(*b*)
- •[4] $(3-x)(1-x)+t=0$
- •[5] $x^2-4x+(3+t)=0$
- •[6] $\Delta=0$ for equal roots or equiv.
- •[7] $\Delta=16-4\times1\times(3+t)$ or equiv.
- •[8] $t=1$

A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC. Coordinate axes have been introduced as shown in the diagram.
The point A divides OP in the ratio 1:2.
Points B and C similarly divide RP and SP respectively in the ratio 1:2.



(*a*) Find the coordinates of A, B and C. **(3)**

(*b*) Calculate the area of triangle ABC. **(4)**

(*c*) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	3	3.1			3				3.1.6		Source
(*b*)	4	3.1			4				3.1.3,	0.1	**1993 Paper 2**
(*c*)	5	0.1			5				0.1		**Qu.5**

(a) •[1] $A(1,0,0)$

•[2] $B(3,2,0)$

•[3] $C(3,0,-2)$

(b) •[4] strategy for area of triangle and attempt to calculate parts

•[5] 60° or altitude $= \sqrt{6}$

•[6] side $= 2\sqrt{2}$

•[7] using chosen formula correctly

(c) •[8] 54 unit^2 for cube

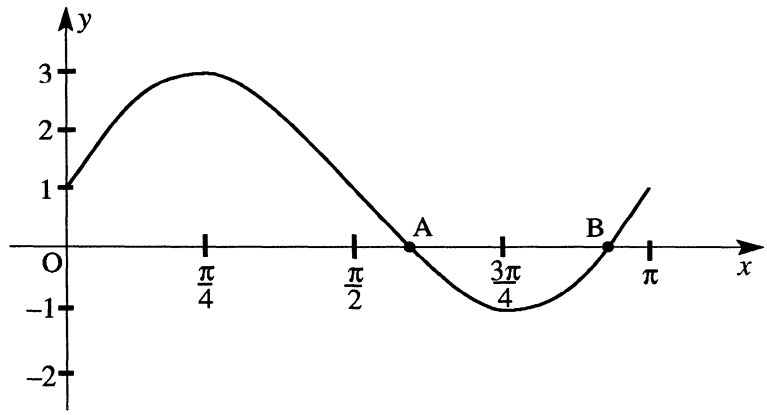
•[9] know how to calculate s.a of crystal

•[10] area of 1 pentagonal face = 7 unit^2

•[11] 51.5 unit^2 for crystal $(48 + 2\sqrt{3})$

•[12] strategy for finding % decrease

The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.



(*a*) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. **(5)**

(*b*) The points (0, 2) and (π, 0) are joined by a straight line *l*. In how many points does *l* intersect the given graph ? **(1)**

(*c*) C is the point on the given graph with an x-coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line *l*. **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.3
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	5	2.3	3	2					2.3.1		Source
(*b*)	1	0.1	1						0.1		**1993 Paper 2**
(*c*)	3	0.1		3					0.1		**Qu.6**

(a) •1 $2\sin 2x + 1 = 0$

•2 $\sin 2x = -\frac{1}{2}$

•3 for any valid sol of equ. in form $\sin ax = -\frac{b}{c}$

•4 $\left(\frac{7\pi}{12}, 0\right)$

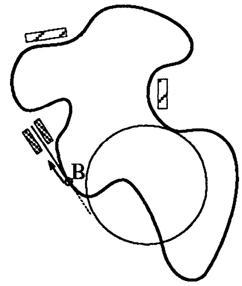
•5 $\left(\frac{11\pi}{12}, 0\right)$

(b) •6 3

(c) •7 $y_C = 1$

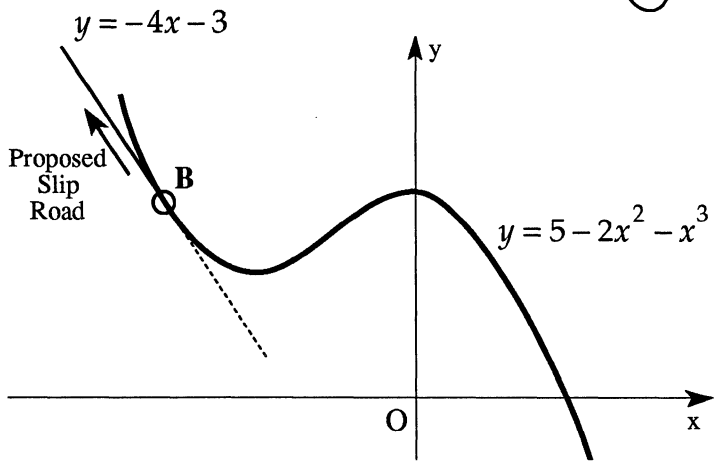
•8 for a strategy to make a decision about C

•9 for making a consistent decision about C



The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.

Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	7	2.1	7						2.1.8,	2.1.3	Source
(*b*)	1	2.1		1					2.1.8		**1993 Paper 2 Qu.7**

(*a*)
- •[1] equating expressions for y
- •[2] re-arranging cubic...... "..."= 0
- •[3] strategy for solving cubic
- •[4] first linear factor
- •[5] quadratic factor
- •[6] $x = -2,\ 2$
- •[7] intersection at (– 2, 5)

(*b*)
- •[8] double root $\Rightarrow$ tangency **or** $y'(-2) = -4$ = gradient of line

Secret Agent 004 has been captured and his captors are giving him a 25 milligram dose of a truth serum every 4 hours. 15% of the truth serum present in his body is lost every hour.

(*a*) Calculate how many milligrams of serum remain in his body after 4 hours (that is immediately **before** the second dose is given). **(3)**

(*b*) It is known that the level of serum in the body has to be continuously above 20 milligrams before the victim starts to confess. Find how many doses are needed before the captors should begin their interrogation. **(3)**

(*c*) Let u_n be the amount of serum (in milligrams) in his body just **after** his n^{th} dose. Show that $u_{n+1} = 0 \cdot 522u_n + 25$. **(1)**

(*d*) It is also known that 55 milligrams of this serum in the body will prove fatal, and the captors wish to keep Agent 004 alive. Is there any maximum length of time for which they can continue to administer this serum and still keep him alive ? **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.4
(*a*)	3	1.4			3				1.4.1		Source
(*b*)	3	1.4			3				1.4.1		**1993 Paper 2**
(*c*)	1	1.4			1				1.4.3		**Qu.8**
(*d*)	4	1.4			3	1			1.4.4,	1.4.5	

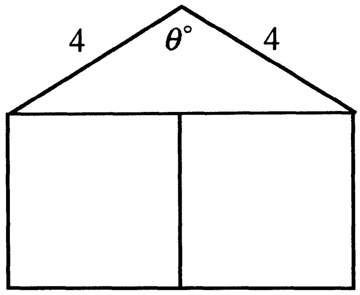
(*a*) •1 strategy for each hour (e.g. using 0·85)
•2 using strategy 4 times (e.g. $(0{\cdot}85^4)$)
•3 13·05

(*b*) •4 apply a correct dose strategy
•5 a relevant sequence e.g. 13·05, 19·86, 23·4,
or 25, 38·05, 44·9, 48·4
•6 3 doses

(*c*) •7 valid explanation i.e. $(0 \cdot 85)^4 = 0 \cdot 522$ explicitly stated

(*d*) •8 statement that limit exists because $(0 \cdot 85)^4 < 1$
•9 $\therefore l = 0 \cdot 522l + 25$ or using $l = \frac{b}{1-a}$
•10 $l = 52{\cdot}3$
•11 52·3 < 55 so no maximum length of time

A builder has obtained a large supply of 4 metre rafters. He wishes to use them to build some holiday chalets. The planning department insists that the gable end of each chalet should be in the form of an isosceles triangle surmounting two squares, as shown in the diagram.



(*a*) If θ ° is the angle shown in the diagram and A is the area (in square metres) of the gable end , show that $A = 8\left(2 + \sin\theta^\circ - 2\cos\theta^\circ\right)$. **(5)**

(*b*) Express $8\sin\theta^\circ - 16\cos\theta^\circ$ in the form $k\sin(\theta - \alpha)^\circ$. **(4)**

(*c*) Find algebraically the value of θ for which the area of the gable end is 30 square metres. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	3.4
(*a*)	5	0.1			1	4			0.1, 2.3.3	Source
(*b*)	4	3.4			4				3.4.1	**1993 Paper 2**
(*c*)	4	3.4			1	3			3.4.2	**Qu.9**

(*a*)
- •[1] area of triangle = $\frac{1}{2} \times 4 \times 4\sin\theta$ *or* $2 \times \frac{1}{2} \times 4\sin\frac{\theta}{2} \times 4\cos\frac{\theta}{2}$
- •[2] strategy for finding length of side of square or rectangle
- •[3] for length of side or (length of side)^2 of square/rectangle
- •[4] area of rectangle
- •[5] simplifying

Note : For •[3] various forms of the length are

square: $4\sin\frac{\theta}{2}, \frac{2\sin\theta}{\sin\left(90 - \frac{\theta}{2}\right)}, \sqrt{16 - 16\cos^2\frac{\theta}{2}}$

rect: $\frac{4\sin\theta}{\sin\left(90 - \frac{\theta}{2}\right)}, \sqrt{32 - 32\cos\theta}$

(*b*)
- •[6] strategy including expansion of $k\sin(\theta - \alpha)$
- •[7] $k\cos\alpha = 8$ & $k\sin\alpha = 16$
- •[8] $k = 8\sqrt{5}$ or equiv.
- •[9] $\tan\alpha = 2 \Rightarrow \alpha = 63\cdot4$

(*c*)
- •[10] $8(2 + \sin\theta - 2\cos\theta) = 30$
- •[11] $8\sqrt{5}\sin(\theta - 63\cdot4)^\circ = 14$
- •[12] $\sin(\theta - 63\cdot4)^\circ = 0\cdot783$
- •[13] $\theta = 51\cdot5 + 63\cdot4 = 114\cdot9$

When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds).This graph is shown in fig. 1.

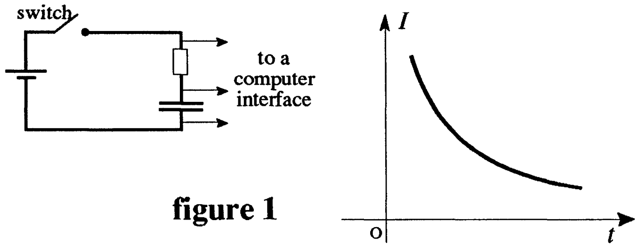


figure 1

In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.

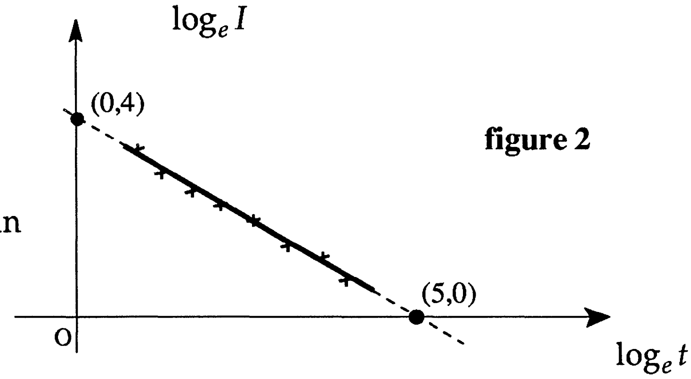


figure 2

(*a*) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. **(3)**

(*b*) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	3	1.1			3				1.1.1,	1.1.7	Source **1993 Paper 2 Qu.10**
(*b*)	4	3.3				4			3.3.6		

(*a*) •[1] $m = -\frac{4}{5}$ stated or implied

•[2] $y = mx + 4$ stated or implied

•[3] $\log_e I = -\frac{4}{5}\log_e t + 4$

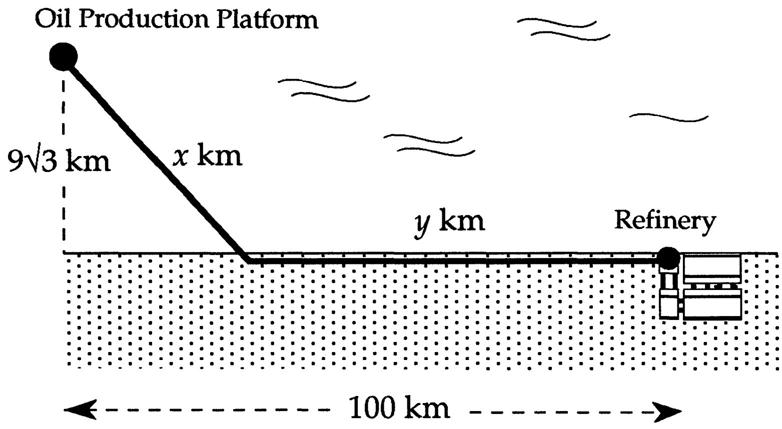
(*b*) •[4] $\log_e t^{-\frac{4}{5}}$

•[5] $\log_e 54.6$

•[6] $\log_e 54.6t^{-\frac{4}{5}}$

•[7] $I = 54.6t^{-0.8}$

An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(*a*) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}. \qquad \textbf{(3)}$$

(*b*) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.2
(*a*)	3	0.1	1	2					0.1		Source
(*b*)	7	1.3	1	6					1.3.15,	3.2.2	**1993 Paper 2 Qu.11**

(*a*) •[1] $C = 2x + y$

•[2] $\sqrt{x^2 - (9\sqrt{3})^2}$

•[3] for completing proof

(*b*) •[4] knowing to differentiate

•[5] $\frac{1}{2}\left(x^2 - 243\right)^{-\frac{1}{2}}$

•[6] $\times\ 2x$

•[7] $C'(18) = 0$

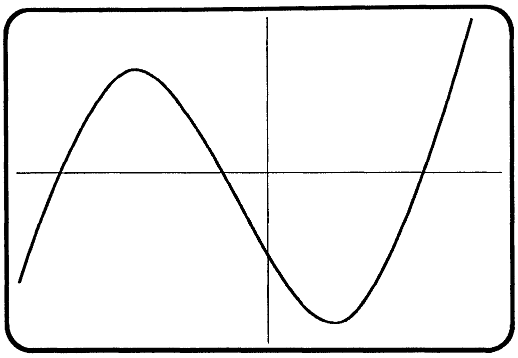
•[8] justification of minimum e.g. nature table

•[9] $C = 127$

•[10] $x + y = 109$

	18^-	18	18^+
$C'(x)$	−	0	+
	╲	—	╱
		minimum	

The diagram shows part of the graph of the curve with equation
$f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**

(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(*c*) Find the turning points and justify their nature. **(6)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	3	2.1	3						2.1.3		Source
(*b*)	2	1.2	2						1.2.9		**1992 Paper 2**
(*c*)	6	1.3	6						1.3.12		**Qu.1**

(*a*)
- •1 any linear factor
- •2 corresponding quadratic factor
- •3 $f(x) = (x+1)(x-4)(x+4)$

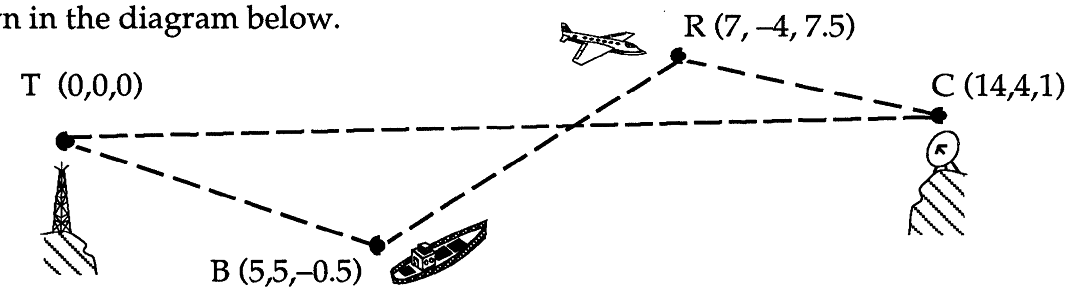
(*b*)
- •4 For all 3 points on x-axis
- •5 (0,–16)

(*c*)
- •6 $f'(x) = 3x^2 + 2x - 16$
- •7 use $f'(x) = 0$
- •8 $x = 2$, **and** $x = -\frac{8}{3}$
- •9 $y = -36$, **and** $y = \frac{400}{27}(14.8)$
- •10

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	–	–	0	+
	⋰	⋯	⋱	⋱	⋯	⋰

- •11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2, -36)$

Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	3	3.1			3				3.1.3		Source
(*b*)	2	3.1			2				3.1.3		**1992 Paper 2**
(*c*)	3	3.1			3				3.1.10		**Qu.2**
(d)	5	3.1			5				3.1.11		

(*a*) •1 Strategy: use vectors or 3-D distance formula

•2 $\overrightarrow{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ *or* $BR^2 = 2^2 + 7^2 + 4^2$

•3 answer

(*b*) •4 $|\overrightarrow{MR}| = \sqrt{115.25}$ or equivalent

•5 answer

(c) •6 know to use a scalar product

•7 $\overrightarrow{TC}.\overrightarrow{BR} = 0$

•8 communication: 0 ⇔ perpendicularity

(*d*) •9 Strategy: know to use

$\cos T\hat{C}R = \dfrac{\overrightarrow{TC}.\overrightarrow{RC}}{|TC||RC|}$ or equiv.

•10 $\overrightarrow{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ and $\overrightarrow{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

•11 $\sqrt{161}$ and $\sqrt{65}$

•12 $\overrightarrow{TC}.\overrightarrow{RC} = 82$

•13 36.7°

Biologists calculate that when the concentration of a particular chemical in a sea loch reaches 5 milligrams per litre (mg/l) the level of pollution endangers the life of the fish.
A factory wishes to release waste containing this chemical into the loch. It is claimed that the discharge will not endanger the fish.
The Local Authority is supplied with the following information:

1. The loch contains none of this chemical at present.
2. The factory manager has applied to discharge effluent once per week which will result in an increase in concentration of 2.5 mg/l of the chemical in the loch.
3. The natural tidal action will remove 40% of the chemical from the loch every week.

(*a*) Show that this level of discharge would result in fish being endangered. **(3)**

When this result is announced, the company agrees to install a cleaning process that reduces the concentration of chemical released into the loch by 30%.

(*b*) Show the calculations you would use to check this revised application.
Should the Local Authority grant permission ? **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.4
(*a*)	3	1.4			3				1.4.3		Source
(*b*)	5	1.4			5				1.4.3,	1.4.5	**1992 Paper 2 Qu.3**

(*a*)
- •1 0.6 stated/implied
- •2 $u_{n+1} = 0.6u_n + 2.5$
- •3 communication: ie 6.25 $\Rightarrow$ danger

(*b*)
- •4 0.7 x 2.5 = 1.75
- •5 2.8, 3.43, 3.808
- •6 $u_{n+1} = 0.6u_n + 1.75$
- •7 limit = 4.375
- •8 communication: ie 4.375 $\Rightarrow$ allow/disallow

(*a*) For a particular radioactive substance the mass m (in grams) at time t (in years) is given by

$$m = m_0 e^{-0.02t}$$

where m_0 is the original mass.

If the original mass is 500 grams, find the mass after 10 years. **(2)**

(*b*) The half-life of any material is the time taken for half of the mass to decay.
Find the half-life of this substance. **(3)**

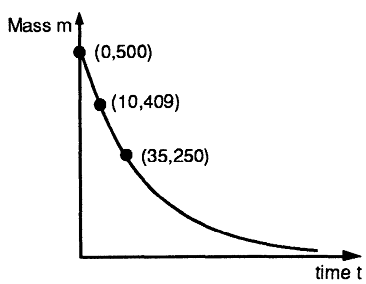
(*c*) Illustrate **ALL** of the above information on a graph. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.3
(*a*)	2	3.3			2				3.3.4		Source
(*b*)	3	3.3			1	2			3.3.4		**1992 Paper 2**
(*c*)	3	1.2			1	2			1.2.5		**Qu.4**

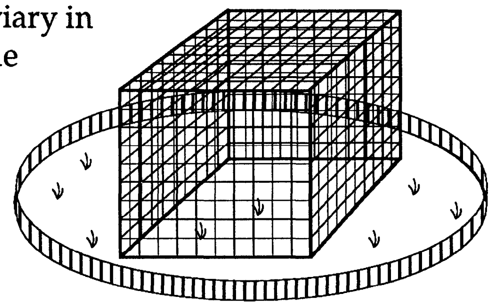
(*a*)
•1 $m = 500e^{-0.02 \times 10}$
•2 409.37 grams

(*b*)
•3 $250 = 500e^{-0.02t}$
•4 $\ln 250 = \ln 500 - 0.02t \times 1$ or equiv.
•5 34.7 years

(*c*)
•6 any **two** of the 3 points
•7 the remaining point
•8 a decreasing curve



The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3.



(*a*) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2000}{x}.$$

(4)

(*b*) Find the dimensions of the aviary to ensure that the cost of netting is minimised. **(6)**

<table>
<tr><td rowspan="2">part marks</td><td rowspan="2">Unit</td><td colspan="2">non-calc</td><td colspan="2">calc</td><td colspan="2">calc neut</td><td>Content Reference :</td><td rowspan="2">1.3</td></tr>
<tr><td>C</td><td>A/B</td><td>C</td><td>A/B</td><td>C</td><td>A/B</td><td>Main Additional</td></tr>
<tr><td>(a) 4
(b) 6</td><td>0.1
1.3</td><td>2
4</td><td>2
2</td><td></td><td></td><td></td><td></td><td>0.1
1.3.15</td><td>Source
1992 Paper 2 Qu.5</td></tr>
</table>

(*a*)
- •1 introduce height specific to this cuboid
- •2 $h = \frac{500}{x^2}$
- •3 $A = x^2 + 4xh$
- •4 $A = x^2 + 4x.\frac{500}{x^2}$ explicitly stated

(*b*)
- •5 $A'(x) = \ldots\ldots$
- •6 $2x - 2000x^{-2}$
- •7 $A'(x) = 0$ specifically stated
- •8 $x = 10$
- •9 justify minimum e.g. with table
- •10 dimensions of 10 by 10 by 5

The *vector product*, $\boldsymbol{a} \times \boldsymbol{b}$, of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined by

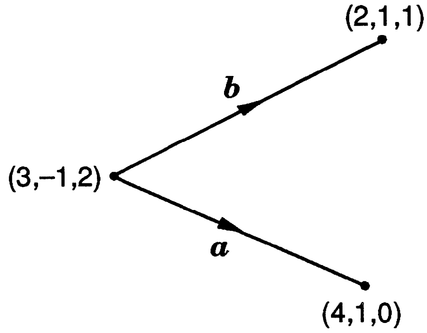
$$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

EXAMPLE

$$\text{when } \boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \text{ then } \boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} 2\times2-3\times0 \\ 3\times(-1)-1\times2 \\ 1\times0-2\times(-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix}$$

(*a*) If $\boldsymbol{a}$ and $\boldsymbol{b}$ are as shown in the diagram and $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$, evaluate $\boldsymbol{c}$. **(3)**

(*b*) By considering $\boldsymbol{a}.\boldsymbol{c}$ and $\boldsymbol{b}.\boldsymbol{c}$, what can be concluded about $\boldsymbol{c}$? **(4)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	3	0.1					3		0.1		Source
(*b*)	4	3.1					4		3.1.9,	3.1.10	1992 Paper 2 Qu.6

(*a*)
- •[1] $a = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$
- •[2] substitute in the rule for a x b
- •[3] answer

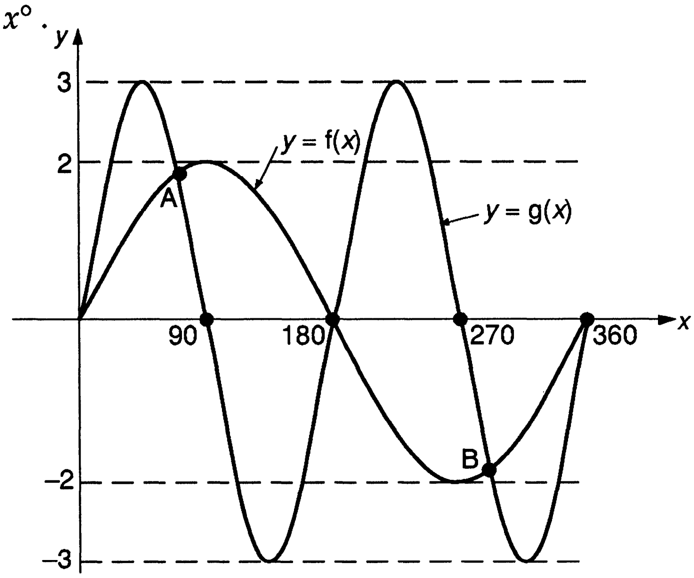
(*b*)
- •[4] evaluate $\underline{a}.\underline{c}$
- •[5] evaluate $\underline{b}.\underline{c}$
- •[6] a statement that $\underline{a}$ is perpendicular to $\underline{c}$
- •[7] a statement that $\underline{b}$ is perpendicular to $\underline{c}$

(*a*) Solve the equation $3\sin 2x° = 2\sin x°$ for $0 \le x \le 360$ **(4)**

(*b*) The diagram below shows parts of the graphs of sine functions *f* and *g*. State expressions for $f(x)$ and $g(x)$. **(1)**

(*c*) Use your answers to part (*a*) to find the co-ordinates of A and B. **(2)**

(*d*) Hence state the values of *x* in the interval $0 \le x \le 360$ for which $3\sin 2x° < 2\sin x°$. **(3)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.3
(*a*)	4	2.3			4				2.3.5	Source
(*b*)	1	1.2			1				1.2.7	**1992 Paper 2**
(*c*)	2	1.2			2				1.2.9	**Qu.7**
(*d*)	3	1.2			2	1			1.2.10	

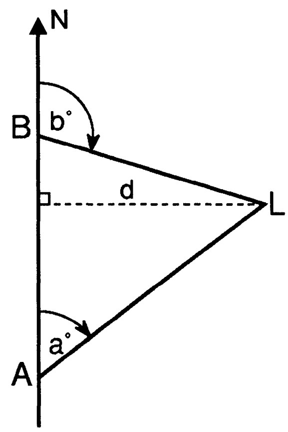
(*a*)
- •1 strategy: ie $\sin 2x = 2\sin x\cos x$
- •2 $\sin x = 0$ AND $\cos x = \frac{1}{3}$
- •3 0, 180 AND 360
- •4 70.5 AND 289.5 AND no other angles

(*b*)
- •5 $f(x) = 2\sin x°,\ g(x) = 3\sin 2x°$

(*c*)
- •6 $x = 70.5$ AND 289.5
- •7 $y = 1.89$ AND −1.89

(*d*)
- •8 70.5 AND 180
- •9 289.5 AND 360
- •10 use inequality signs logically to connect the points of intersection (ie **not** for $180 < x < 70.5$)

A ship is sailing due north at a constant speed. When at position A, lighthouse L is observed on a bearing of a°. One hour later, when the ship is at position B, the lighthouse is on a bearing of b°. The shortest distance between the ship and the lighthouse during this hour was d miles.



(*a*) Prove that $AB = \frac{d}{\tan a^\circ} - \frac{d}{\tan b^\circ}$. **(2)**

(*b*) Hence prove that $AB = \frac{d\sin(b-a)^\circ}{\sin a^\circ \sin b^\circ}$. **(3)**

(*c*) Calculate the shortest distance from the ship to the lighthouse when the bearings a° and b° are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(*a*)	2	0.1			1	1			0.1		Source
(*b*)	3	2.3				3			2.3.4		**1992 Paper 2**
(*c*)	3	0.1			3				0.1		**Qu.8**

(*a*) •1 $CA = \frac{d}{\tan a}$

•2 $CB = \frac{d}{\tan(180-b)}$

(*b*) •3 $AB = \frac{d}{\frac{\sin a}{\cos a}} - \frac{d}{\frac{\sin b}{\cos b}}$

•4 $\frac{d\cos a}{\sin a} - \frac{d\cos b}{\sin b}$

•5 $\frac{d\sin b\cos a - d\cos b\sin a}{\sin a\sin b}$

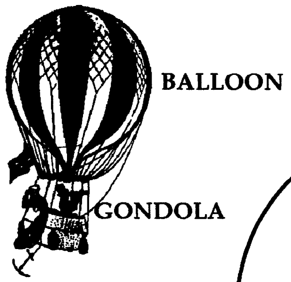
(*c*) •6 AB = 14

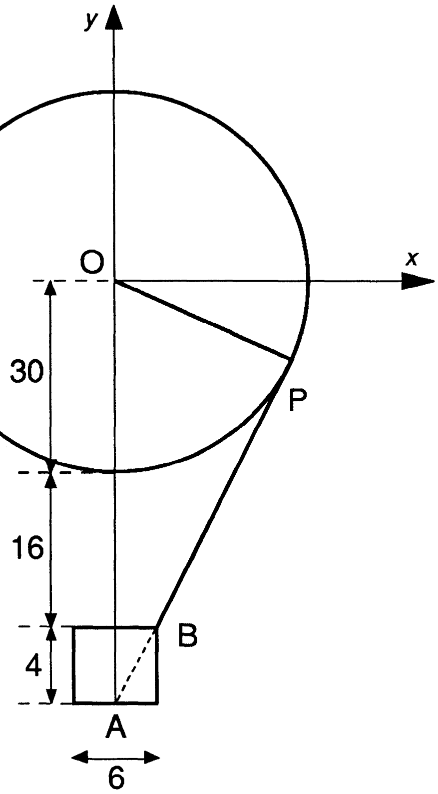
•7 1.577 or 0.634

(comes from AB = 1.577d or d = 0.634 AB)

•8 8.9 miles

A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.





Co-ordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.

(*a*) Find the equation of the cable PB. **(3)**

(*b*) State the equation of the circle representing the balloon. **(1)**

(*c*) Prove that this cable is a tangent to the balloon and find the co-ordinates of the point P. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.4
(*a*)	3	1.1					3		1.1.1,	1.1.7	Source
(*b*)	1	2.4					1		2.4.3		**1992 Paper 2**
(*c*)	5	2.4					2	3	2.4.4		**Qu.9**

(*a*) •1 Strategy: know to find m

•2 $m = \frac{4}{3}$

•3 $y + 46 = \frac{4}{3}(x - 3)$

(*b*) •4 $x^2 + y^2 = 900$ or equivalent

(*c*) •5 Strategy: know to substitute

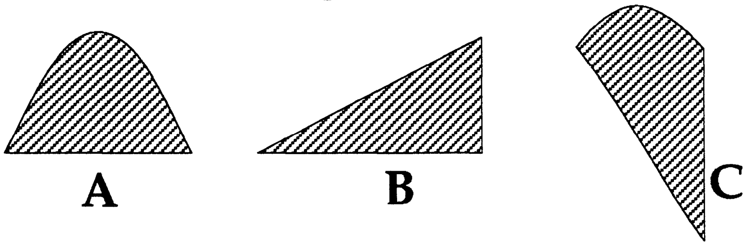
•6 $x^2 + \left(\frac{4}{3}x - 50\right)^2 = 900$

•7 $(x - 24)^2$ or evaluate the discriminant

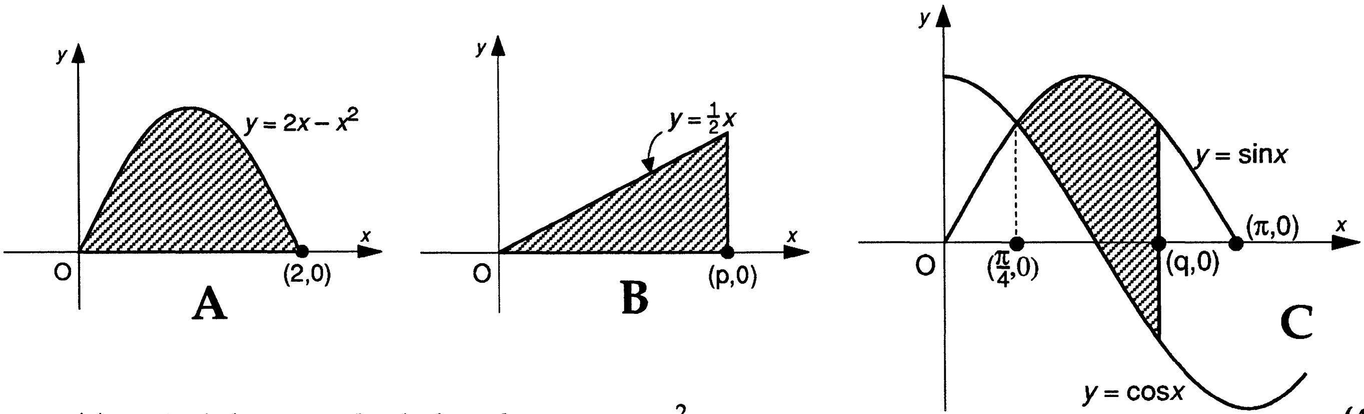
•8 communication relating to tangency

•9 $(24, -18)$

An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the **same** area. Three of the shapes used are drawn below.



Relative to x,y–axes, the shapes are positioned as shown below.



(*a*) Find the area shaded under $y = 2x - x^2$. **(4)**

(*b*) Use the area found in part (*a*) to find the value of p. **(2)**

(c) Prove that q satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of q to 2 significant figures. **(10)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.4
(*a*)	4	2.2			4				2.2.6		Source
(*b*)	2	0.1			2				0.1		**1992 Paper 2**
(*c*)	10	3.4			2	8			3.4.2,	3.2.1, 2.2.7	**Qu.10**

(*a*) •1 strategy: know to integrate

•2 $\int_0^2 \left(2x - x^2\right)\, dx$

•3 $x^2 - \frac{1}{3}x^3$

•4 $1\frac{1}{3}$ $units^2$

(*b*) •5 strategy: use area to find p

•6 $p = \frac{4}{\sqrt{3}}$ or equivalent

(*c*) •7 $\int (\sin x - \cos x)\, dx$

•8 for the limits $\int_{\frac{\pi}{4}}^{q}$

•9 $[-\cos x - \sin x]$

•10 $-\cos q - \sin q + \sqrt{2}$

•11 $\sqrt{2} - \frac{4}{3} = 0.081$

•12 strategy: eg $k\cos(q - \alpha)$

•13 $k = \sqrt{2}$

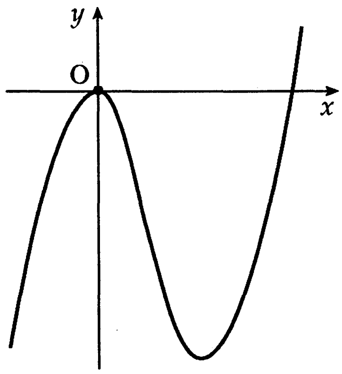
•14 $\alpha = \frac{\pi}{4}$

•15 $\cos\left(q - \frac{\pi}{4}\right) = \frac{0.081}{\sqrt{2}}$

•16 $q = 2.3$

(*a*) The diagram shows a part of the curve with equation $y = 2x^2(x-3)$. Find the coordinates of the stationary points on the graph and determine their nature. **(5)**

(*b*) State the range of values of k for which $y = k$ intersects the graph in three distinct points. **(2)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	5	1.3					5		1.3.12		Source 1991 Paper 2 Qu. 1
(*b*)	2	1.2					2		1.2.1		

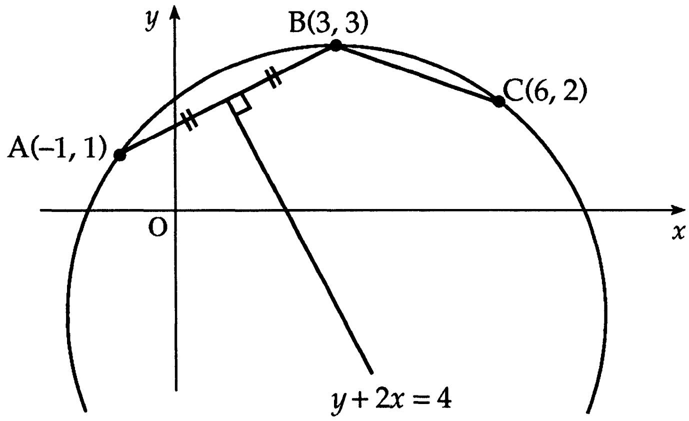
(*a*)
- •[1] $\frac{dy}{dx} = 6x^2 - 12x$
- •[2] $\frac{dy}{dx} = 0$
- •[3] $x = 0,\ x = 2$
- •[4]

x	0^-	0	0^+	2^-	2	2^+
$\frac{dy}{dx}$	+	−	+	+	−	+

- •[5] max. at (0,0) min at (2,–8)

(*b*)
- •[6] $k < 0$
- •[7] $k > -8$

(*a*) In the diagram, A is the point (–1, 1), B is (3, 3) and C is (6, 2). The perpendicular bisector of AB has equation $y + 2x = 4$. Find the equation of the perpendicular bisector of BC. **(4)**



(*b*) Find the centre and the equation of the circle which passes through A, B and C. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	4	1.1					4		1.1.9,	1.1.7	Source: **1991 Paper 2 Qu. 2**
(*b*)	6	2.4					6		2.4.3,	1.1.2	

(*a*) •[1] $m_{BC} = -\frac{1}{3}$

•[2] $m_{\perp} = 3$

•[3] $\text{midpoint}_{BC} = \left(\frac{9}{2}, \frac{5}{2}\right)$

•[4] $y - \frac{5}{2} = 3\left(x - \frac{9}{2}\right)$

(*b*) •[5] $y - 3x = -11$

•[6] perp. bisector passes thr' centre **stated explicitly**

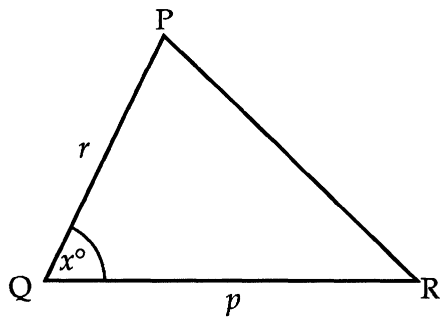
•[7] using $y - 3x = -11$ **and** $y + 2x = 4$

•[8] $(3, -2)$

•[9] $r^2 = 25$

•[10] $(x-3)^2 + (y+2)^2 = 25$

The diagram shows an isosceles triangle PQR in which PR = QR and angle PQR = $x°$.



(*a*) Show that $\frac{\sin x°}{p} = \frac{\sin 2x°}{r}$. **(3)**

(*b*) (i) State the value of $x°$ when $p = r$.

(ii) Using the fact that $p = r$, solve the equation in (*a*) above, to justify your stated value of $x°$. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(*a*)	3	0.1					2	1	0.1		Source
(*b*)	5	2.3					5		2.3.5,	0.1	**1991 Paper 2 Qu. 3**

(*a*) •1 $(180 - 2x)°$

•2 $\frac{\sin x°}{p} = \frac{\sin(180 - 2x)°}{r}$

•3 $\sin(180 - 2x)° = \sin 2x°$ **stated explicitly**

(*b*) •4 $60°$

•5 $\sin x° = \sin 2x°$

•6 $\sin x°(2\cos x° - 1) = 0$

•7 $\sin x° = 0$ *and* $\cos x° = \frac{1}{2}$

•8 $x = 60$ is only answer **stated explicitly**

(*a*) On the same diagram, sketch the graphs of $y = \log_{10} x$ and $y = 2 - x$ where $0 < x < 5$.
Write down an approximation for the x-coordinate of the point of intersection. **(3)**

(*b*) Find the value of this x-coordinate, correct to 2 decimal places. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(*a*)	3	1.2			3				1.2.5		Source
(*b*)	3	0.1			1	2			0.1		1991 Paper 2 Qu. 4

(*a*) •[1] graph of $y = 2 - x$ with two annotated points eg (2,0) and (0,2)
•[2] graph of $y = \log_{10} x$ with one annotated point eg (1,0)
•[3] any consistent approximation

(*b*) •[4] between 1.7 and 1.8
•[5] between 1.75 and 1.80
•[6] 1.76

Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

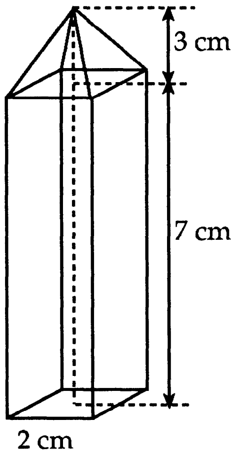


Diagram 1

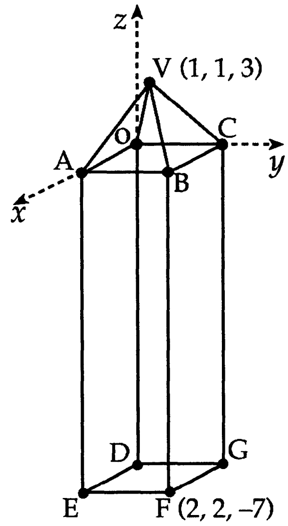


Diagram 2

(*a*) Find

(i) the components of the vectors represented by $\vec{VF}$ and $\vec{VE}$;

(ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.

Calculate the area of the glass triangle VEF. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.1
(*a*)	7	3.1					7		3.1.11,	3.1.1	Source **1991 Paper 2 Qu. 5**
(*b*)	3	0.1					3		0.1		

(*a*)

•[1] $\vec{VF} = \begin{pmatrix} 1 \\ 1 \\ -10 \end{pmatrix}$

•[2] E = (2,0,–7)

•[3] $\vec{VE} = \begin{pmatrix} 1 \\ -1 \\ -10 \end{pmatrix}$

•[4] $\cos E\hat{V}F = \frac{\vec{VE}.\vec{VF}}{|\vec{VE}|\,|\vec{VF}|}$ This may appear as $\frac{100}{102}$ **after the completion of •[5] and •[6].**

•[5] $\vec{VE}.\vec{VF} = 100$

•[6] $|\vec{VE}|\,|\vec{VF}| = 102$

•[7] 11.4°

(*b*)

•[8] $\frac{1}{2} VE \times VF \sin E\hat{V}F$

•[9] $\frac{1}{2} \times 102 \times \sin 11.4°$

•[10] 10.02

There is a rule known as the Product Rule which is used, as shown below, to differentiate any product of two functions of the same variable.

The Product Rule

If $P(x) = f(x).g(x)$, then $P'(x) = f'(x).g(x) + f(x).g'(x)$

Example: Find the derivative of $P(x) = x^2 \sin x$.

$P(x) = x^2 \sin x$ Choose $f(x) = x^2$ *and* $g(x) = \sin^2 x$

then $f'(x) = 2x$ *and* $g'(x) = \cos x$

so $P'(x) = 2x.\sin x + x^2.\cos x$

$P'(x) = 2x \sin x + x^2 \cos x$

Use the Product Rule to find the derivative of $P(x) = x^3 \cos x$ **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	0.1
(-)	5	0.1					5		0.1		Source **1991 Paper 2 Qu. 6**

(-) •[1] $f(x) = x^3$

•[2] $g(x) = \cos x$

•[3] $f'(x) = 3x^2$ **and** $g(x) = -\sin x$

•[4] $3x^2 \cos x$

•[5] $-x^3 \sin x$

(*a*) A tractor tyre is inflated to a pressure of 50 units.
Twenty-four hours later the pressure has dropped to 10 units.

If the pressure, P_t units, after t hours is given by the formula $P_t = P_0 e^{-kt}$, find the value of k, to three decimal places. **(5)**

(*b*) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre? **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	5	3.3			2	3			3.3.4		Source
(*b*)	4	3.3			1	3			3.3.4		**1991 Paper 2 Qu. 7**

(*a*)
- •[1] $10 = 50e^{-24k}$
- •[2] $0.2 = e^{-24k}$
- •[3] $-24k = \ln 0.2$
- •[4] $-24k = -1.609$
- •[5] $k = 0.067$

(*b*)
- •[6] knowing to find P_4
- •[7] $P_4 = 50e^{-0.067 \times 4}$
- •[8] 38
- •[9] $38 > 30$ so can be driven further

The displacement, d units, of a wave after t seconds, is given by the formula $d = \cos 20t° + \sqrt{3}\sin 20t°$.

(*a*) Express d in the form $k\cos(20t - \alpha)°$, where $k > 0$ and $0 \leq \alpha \leq 360$. **(4)**

(*b*) Sketch the graph of d for $0 \leq t \leq 18$. **(4)**

(*c*) Find, correct to 1 decimal place, the values of t, $0 \leq t \leq 18$, for which the displacement is 1·5 units. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
(*a*)	4	3.4			4				3.4.1		Source
(*b*)	4	1.2			2	2			1.2.3		**1991 Paper 2**
(*c*)	3	2.3			1	2			2.3.1		**Qu. 8**

(*a*)
- •1 $k\cos 20t°\cos\alpha° + k\sin 20t°\sin\alpha°$
- •2 $k\cos\alpha° = 1$ *and* $k\sin\alpha° = \sqrt{3}$
- •3 $k = 2$
- •4 $\alpha = 60$

(*b*)
- •5 endpoints: $(0,1)$ **or** $(18,1)$
- •6 zeros: $(7.5,0)$ **and** $(16.5,0)$
- •7 stationary points: $(3,2)$ **and** $(12,-2)$
- •8 correct annotation of graph

(*c*)
- •9 $2\cos(20t - 60)° = 1.5$
- •10 $20t - 60 = 41.4 \Rightarrow t = 5.1$
- •11 $20t - 60 = -41.4 \Rightarrow t = 0.9$

(*a*) At 12 noon a hospital patient is given a pill containing 50 units of antibiotic.
By 1 pm the number of units in the patient's body has dropped by 12%.
By 2 pm a further 12% of the units remaining in the body at 1 pm is lost.
If this fall-off rate is maintained, find the number of units of antibiotic remaining at 6 pm. **(4)**

(*b*) A doctor considers prescribing a course of treatment which involves a patient taking one of these pills every 6 hours over a long period of time.
The doctor knows that mopre than 100 units of this antibiotic in the body is regarded as too dangerous.
Should the doctor prescribe this course of treatment?
Give reasons for your answer. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.4
											Source
(*a*)	4	1.4			4				1.4.1		1991 Paper 2
(*b*)	6	1.4			4	2			1.4.3, 1.4.5		Qu. 9

(*a*)
- •[1] use 0.88 or 88%
- •[2] $n = 6$
- •[3] $u_6 = 50 \times 0.88^6$
- •[4] 23.22

(*b*)
- •[5] adding 50
- •[6] $u_{n+1} = 0.88^6 u_n + 50$
- •[7] $-1 < 0.88^6$ (*or* $0.4644) < 1$ so limit exists
- •[8] $L = \frac{50}{1 - 0.88^6}$
- •[9] 93.4
- •[10] $93.4 < 100$ so safe to continue

Diagram 1 shows a rectangular plate of transparent plastic moulded into a parabolic shape and pegged to the ground to form a cover for growing plants. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.
Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy. (All dimensions are given in centimetres.)

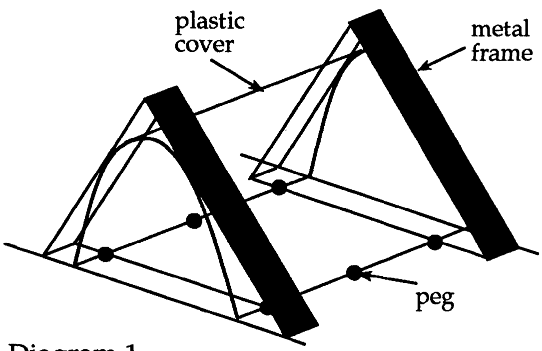


Diagram 1

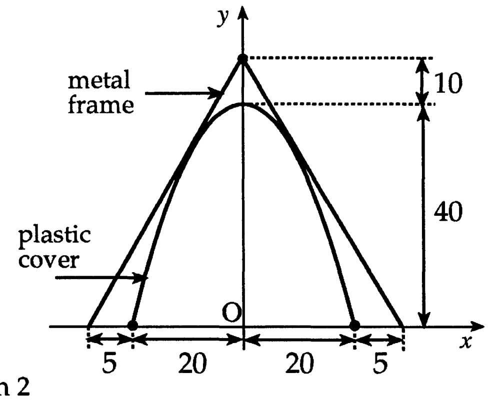


Diagram 2

(*a*) Show that the equation of the parabolic end is $y = 40 - \frac{x^2}{100}$, $-20 \le x \le 20$. **(4)**

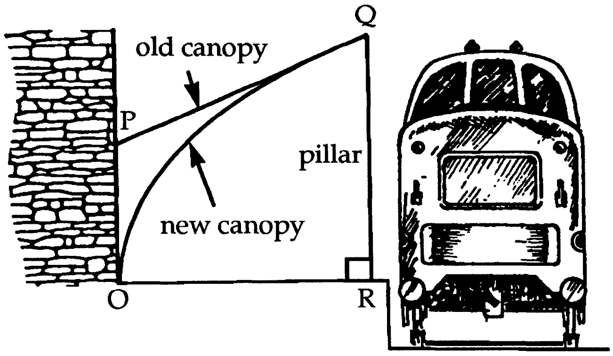
(*b*) Show that the triangular frame touches the cover without disturbing the parabolic shape. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.1
(*a*)	4	1.2					1	3	1.2.7		Source
(*b*)	7	2.1					3	4	2.1.8,	1.1.1, 1.1.7	**1991 Paper 2 Qu. 10**

(*a*)
- •[1] $y = ax^2 + bx + c$
- •[2] $(0,40) \Rightarrow c = 40$
- •[3] symmetry $\Rightarrow b = 0$
- •[4] $(20,0) \Rightarrow a = -\frac{1}{10}$

(*b*)
- •[5] strategy: find equ of line and solve with parabola
- •[6] *e.g.* gradient of left line = 2
- •[7] $y = 2x + 50$
- •[8] $2x + 50 = 40 - \frac{1}{10}x^2$
- •[9] $x^2 + 20x + 100 = 0$
- •[10] $b^2 - 4ac = 0$ *or* $(x-10)^2 = 0$
- •[11] equal roots so line is tangent to parabola

The diagram shows a proposed replacement of the old platform canopy at the local railway station by a new parabolic canopy, while keeping the original pillars.
If OR and OP are taken as the x- and y- axes and Q has coordinates (1, 1), then OQ has equation $y = \sqrt{x}$ and PQ is the tangent at Q to the parabola.



The planners have received an objection that there is a reduction of more than 10% in the space under the canopy and wish to compare the two canopies.

(*a*) Find the equation of the tangent PQ and the coordinates of P. **(5)**

(*b*) Find the area of the trapezium OPQR. **(2)**

(*c*) Find the area under the parabola OQ. **(3)**

(*d*) Comment on the objection received. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.2
(*a*)	5	1.3	5						1.3.9,	1.1.7	Source
(*b*)	2	0.1	2						0.1		**1991 Paper 2**
(*c*)	3	2.2	3						2.2.6		**Qu. 11**
(*d*)	3	0.1	1	2					0.1		

(*a*)
- •[1] $\frac{dy}{dx} = \ldots\ldots$
- •[2] $\frac{1}{2}x^{-\frac{1}{2}}$
- •[3] $m = \frac{dy}{dx}_{x=1} = \frac{1}{2}$
- •[4] $y - 1 = \frac{1}{2}(x - 1)$
- •[5] $\text{P} = (0, \frac{1}{2})$

(*b*)
- •[6] method for area of trapezium
- •[7] $\frac{3}{4}$

(*c*)
- •[8] $\int_0^1 x^{\frac{1}{2}}\, dx$
- •[9] $\frac{2}{3}x^{\frac{3}{2}}$
- •[10] $\frac{2}{3}$

(*d*)
- •[11] strategy: compare reduction with original
- •[12] $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$ *and* $\frac{\frac{1}{12}}{\frac{3}{4}} = \frac{1}{9}$
- •[13] $\frac{1}{9} = 11.1\% > 10\%$ so objection correct

A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbf{R}$.

(*a*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **(3)**

(*b*) Find the stationary points of this curve $y = f(x)$ and determine their nature. **(7)**

(*c*) Sketch the curve $y = f(x)$. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	1.3
(*a*)	3	1.2	3						1.2.9	Source
(*b*)	7	1.3	7						1.3.15	**1990 Paper 2**
(*c*)	2	1.3	2						1.3.13	**Qu. 1**

(*a*)
- •1 $x = 1, -2$
- •2 $(1,0)$ **and** $(-2,0)$
- •3 $(0,2)$

(*b*)
- •4 $f(x) = x^3 - 3x + 2$
- •5 $f'(x) = 3x^2 - 3$
- •6 $f'(x) = 0$ **stated explicitly**
- •7 $x = 1$ **and** -1
- •8

x	-1^-	-1	-1^+	1^-	1	1^+
$f'(x)$	+	0	−	−	0	+

- •9 max at $(-1,4)$
- •10 min at $(1,0)$

(*c*)
- •11 correct shape of sketch
- •12 correct annotation of sketch(max, min, 2 axes intersections)

P, Q and R have coordinates (1, –2), (6, 3) and (9, 14) respectively and are three vertices of a kite PQRS.

(*a*) Find the equations of the diagonals of this kite and the coordinates of the point where they intersect. **(7)**

(*b*) Find the coordinates of the fourth vertex S. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.1
(*a*)	7	1.1					7		1.1.10,	0.1	Source **1990 Paper 2 Qu. 2**
(*b*)	2	0.1					2		0.1		

(*a*)
- •1 $m_{PR} = 2$
- •2 PR: *e.g.* $y + 2 = 2(x - 1)$
- •3 knowing to use $m_1 m_2 = 1$ for m_{QS}
- •4 $m_{QS} = -\frac{1}{2}$
- •5 QS: *e.g.* $y - 3 = -\frac{1}{2}(x - 6)$
- •6 knowing to solve simultaneously
- •7 $S = (4,4)$

(*b*)
- •8 $\vec{QM} = \vec{MS}$ or equivalent indication
- •9 $S = (2,5)$

The extract below is taken from the "OIL RIG NEWS".

> **RARE ILLNESS STRIKES RIG**
> **Storm prevents delivery of medicine**
> By noon on Tuesday 20th December 1988 50 of our oil rig personnel were laid low by a mystery illness.
> Our resident medical officer is expressing concern because the number of personnel affected is increasing each day by 8% of the previous day's total.

(*a*) If the daily rate of increase remained at 8% of the previous day's total, how many personnel were affected by noon on Sunday 25th December 1988? **(3)**

(*b*) An improvement in the weather conditions allowed a team of medics to fly out to the rig on the morning of Tuesday 27th December 1988.
At noon on that Tuesday, all personnel were innoculated and no new cases of the illness arose. Within the next 24 hours, 21% of those who had been affected had recovered.
If the daily rate of recovery of 21% of the previous day's total was maintained, how many personnel were still affected by the illness at noon on Saturday 31st December 1988? **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.4
(*a*)	3	1.4			3				1.4.2		Source
(*b*)	5	1.4			5				1.4.2		**1990 Paper 2 Qu. 3**

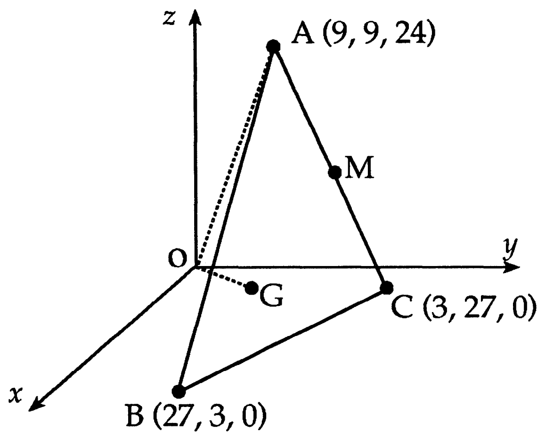
(*a*)
- •[1] $u_n = 1.08^n u_0$
- •[2] $u_5 = 1.08^5 \times 50$
- •[3] 73 or 74

(*b*)
- •[4] $u_7 = 1.08^7 \times 50$
- •[5] $u_7 = 85$ or 86
- •[6] $v_n = 0.79^n v_0$
- •[7] $v_4 = 33$ or 34
- •[8] for consistent rounding

(*a*) Relative to mutually perpendicular axes Ox, Oy and Oz, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.

Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

(*b*) Calculate the size of angle GOA. **(5)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	3.1
(*a*)	3	3.1			3				3.1.6	Source
(*b*)	5	3.1			5				3.1.11	**1990 Paper 2 Qu. 4**

(*a*) •1 $M = (6,\ 18,\ 12)$

•2 e.g. $\vec{BG} = \frac{2}{3}\begin{pmatrix}-21\\15\\12\end{pmatrix}$

•3 $G = (13,\ 13,\ 8)$

(*b*) •4 $\cos A\hat{O}G = \dfrac{\vec{OA}.\vec{OG}}{|\vec{OA}|\,|\vec{OG}|}$

•5 $\vec{OA} = \begin{pmatrix}9\\9\\24\end{pmatrix}$ **and** $\vec{OG} = \begin{pmatrix}13\\13\\8\end{pmatrix}$

•6 $\vec{OA}.\vec{OG} = 426$

•7 $|\vec{OA}| = \sqrt{738}$ **and** $|\vec{OG}| = \sqrt{402}$

•8 $38.5°$

(*a*) Show that $2\cos(x+30)° - \sin x°$ can be written as $\sqrt{3}\cos x° - 2\sin x°$. **(3)**

(*b*) Express $\sqrt{3}\cos x° - 2\sin x°$ in the form $k\cos(x+\alpha)°$ where $k > 0$ and $0 \le \alpha \le 360$ and find the values of k and α. **(4)**

(*c*) Hence, or otherwise, solve the equation $2\cos(x+30)° = \sin x° + 1$, $0 \le x \le 360$. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
(*a*)	3	2.3			3				2.3.2,	1.2.11	Source
(*b*)	4	3.4			4				3.4.1		1990 Paper 2
(*c*)	3	3.4				3			3.4.2		Qu. 5

(*a*)

•1 $\cos(x+30)° = \cos x° \cos 30° - \sin x° \sin 30°$

•2 $\frac{\sqrt{3}}{2}\cos x° - \frac{1}{2}\sin x°$

•3 $2 \times \left(\frac{\sqrt{3}}{2}\cos x° - \frac{1}{2}\sin x°\right) - \sin x°$

(*b*)

•4 $k\cos x° \cos\alpha° - k\sin x° \sin\alpha°$

•5 $k\sin\alpha° = \sqrt{3}$ **and** $k\sin\alpha° = 1$

•6 $k = \sqrt{7}\,\overrightarrow{OG} = 426$

•7 $\alpha = 49.1$

(*c*)

•8 $\sqrt{7}\cos(x+49.1)° = 1$

•9 $x = 18.7°$

•10 $x = 243.1°$

(*a*) The function *f* is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.

The function *g* is defined by $g(x) = x - 1$.

Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$. **(4)**

(*b*) Factorise fully $f(g(x))$. **(3)**

(*c*) The function *k* is such that $k(x) = \dfrac{1}{f(g(x))}$.

For what values of *x* is the function *k* not defined? **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.1
(*a*)	4	1.2	4						1.2.6	Source
(*b*)	3	2.1	3						2.1.3	**1990 Paper 2**
(*c*)	2	1.2	2						1.2.1	**Qu. 6**

(*a*) •1 $f(g(x)) = f(x-1)$

•2 $(x-1)^3 - 2(x-1)^2 - 5(x-1) + 6$

•3 $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

•4 $-2x^2 + 4x - 2 - 5x + 5 + 6$ and completing argument

(*b*) •5 *first* "0" *e.g.*

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

•6 $x^2 - 3x - 4 = (x+1)(x-4)$

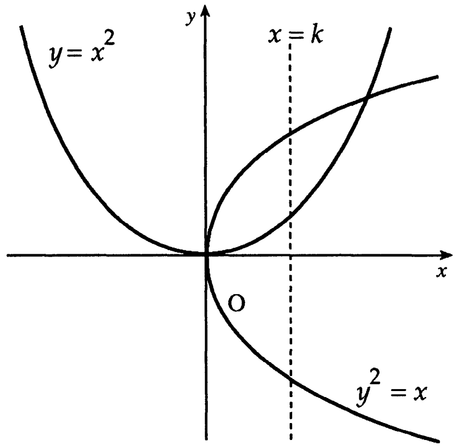
•7 $(x-2)(x+1)(x-4)$

(*c*) •8 denominator $(= (x-2)(x+1)(x-4)) \neq 0$

•9 $-1,\ 2,\ 4$

The diagram shows two curves with equations $y = x^2$ and $y^2 = x$.

The area completely enclosed between the two curves is divided in half by the line with equation $x = k$.



(*a*) Represent these two equal areas by two separate integrals each involving *k*. **(6)**

(*b*) Equate the integrals and show that *k* is given by the equation

$$2k^3 - 4k^{\frac{3}{2}} + 1 = 0.$$ **(4)**

(*c*) Use the substitution p^2 for k^3 to find the value of *k*. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	6	2.2	3	3					2.2.7		Source
(*b*)	4	2.2	2	2					2.2.5,	0.1	1990 Paper 2
(*c*)	4	0.1		4					0.1		Qu. 7

(*a*)
- •1 strategy: equate functions
- •2 $x^4 = x$
- •3 $x(x^3 - 1) = 0 \Rightarrow x = 0,\ x = 1$
- •4 $\int x^{\frac{1}{2}} - x^2\ dx$
- •5 $\int_0^k x^{\frac{1}{2}} - x^2\ dx$
- •6 $\int_k^1 x^{\frac{1}{2}} - x^2\ dx$

(*b*)
- •7 $\frac{2}{3}x^{\frac{3}{2}}$
- •8 $\frac{1}{3}x^3$
- •9 $\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3$ *or* $\frac{2}{3} - \frac{1}{3} - \left(\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3\right)$
- •10 $\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3 = \frac{2}{3} - \frac{1}{3} - \left(\frac{2}{3}k^{\frac{3}{2}} - \frac{1}{3}k^3\right)$ and completing proof

(*c*)
- •11 $2p^2 - 4p + 1 = 0$
- •12 strategy for solving: *e.g.* $p = \dfrac{4 \pm \sqrt{16-8}}{4}$
- •13 $p = 0.293,\ 1.707$
- •14 $k = 0.441$

A sports club awards trophies in the form of paperweights bearing the club crest.
Diagram 1 shows the front view of one of these paperweights. Each is made from two different types of glass. The two circles are concentric and the base line is a tangent to the inner circle.

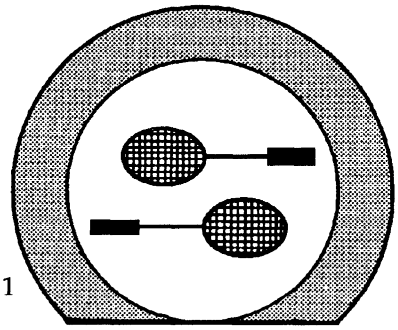


Diagram 1

(*a*) Relative to *x*, *y* coordinate axes, the equation of the outer circle is $x^2 + y^2 - 8x + 2y - 19 = 0$ and the equation of the base line is $y = -6$.

Show that the equation of the inner circle is $x^2 + y^2 - 8x + 2y - 8 = 0$. **(4)**

(*b*) An alternative form of the paperweight is made by cutting off a piece of glass from the original design along a second line with equation $3x - 4y + 9 = 0$ as shown in diagram 2.
Show that this line is a tangent to the inner circle and state the coordinates of the point of contact. **(7)**

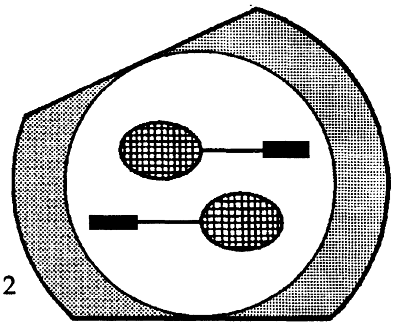


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.4
(*a*)	4	2.4					4		2.4.3		Source
(*b*)	7	2.4					3	4	2.4.4		**1990 Paper 2 Qu. 8**

(*a*)
- •[1] centre = (4,–1)
- •[2] inner radius = 5
- •[3] $(x-4)^2 + (y+1)^2 = 25$
- •[4] completing argument

(*b*)
- •[5] e.g. $x = \frac{4}{3}y - 3$
- •[6] $\left(\frac{4}{3}y - 3\right)^2 + y^2 - 8\left(\frac{4}{3}y - 3\right) + 2y - 8 = 0$
- •[7] $\frac{16}{9}y^2 - 8y + 9 + y^2 - \frac{32}{3}y + 24 + 2y - 8$
- •[8] $y^2 - 6y + 9 = 0$
- •[9] e.g. $(y-3)(y-3) = 0$
- •[10] equal roots ⇒ line is a tangent
- •[11] (1,3)

Polynomial equations often have roots which are not whole numbers.
One method of estimating the roots of such equations is to make repeated use of the following:

> If $x = p$ is an estimate of a root of the equation $f(x) = 0$, then $x = q$ will will be a closer estimate where $q = p - \frac{f(p)}{f'(p)}$.

> **Example**
>
> One of the roots of the equation $x^2 - 2x - 5 = 0$ is known to lie between 3 and 4.
>
> We have $f(x) = x^2 - 2x - 5$ and so $f'(x) = 2x - 2$.
>
> **Choose p = 3** (1st estimate) then $q = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{-2}{4} = 3.5$.
>
> **Choose p = 3.5** (2nd estimate) then $q = 3.5 - \frac{f(3.5)}{f'(3.5)} = 3.5 - \frac{0.25}{5} = 3.45$.
>
> **Choose p = 3.45** (3rd estimate) then $q = 3.45 - \frac{f(3.45)}{f'(3.45)} = 3.45 - \frac{0.0025}{4.9} = 3.449$.
>
> **Conclusion** The root, correct to 1 decimal place, is x = **3.4**

(*a*) Show that the equation $x^3 - 2x^2 + 6x - 4 = 0$ has a root between 0 and 1. **(3)**

(*b*) Use the method described above to find this root correct to 1 decimal place. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	4
											Source
(*a*)	3	2.1			3				2.1.11		1990 Paper 2
(*b*)	6	0.1			3	3			0.1		Qu. 9

(*a*)
- •1 $0^3 - 2\times0 + 6\times0 - 4 = -4$
- •2 $1^3 - 2\times1 + 6\times1 - 4 = 1$
- •3 $f(0) < 0$ and $f(1) > 0$ so $0 < \text{root} < 1$

(*b*)
- •4 $f'(x) = 3x^2 - 4x + 6$
- •5 *e.g.* 1st est = 0, 2nd est $= 0 - \frac{f(0)}{f'(0)} = 0.67$
- •6 3rd est $= 0.67 - \frac{f(0.67)}{f'(0.67)}$
- •7 0.7936
- •8 4th est $= 0.7936 - \frac{f(0.7936)}{f'(0.7936)} = 0.7932$
- •9 0.8

The Water Board of a local authority discovered it was able to represent the approximate amount of water $W(t)$, in millions of gallons, stored in a reservoir t months after the 1st May 1988 by the formula $W(t) = 1.1 - \sin\frac{\pi t}{6}$.

The board then predicted that under normal conditions this formula would apply for three years.

(*a*) Draw and label sketches of the graphs of $y = \sin\frac{\pi t}{6}$ and $y = -\sin\frac{\pi t}{6}$, for $0 \le t \le 36$ on the same diagram. **(4)**

(*b*) On a separate diagram and using the same scale on the t-axis as you used in part (*a*), draw a sketch of the graph of $W(t) = 1.1 - \sin\frac{\pi t}{6}$. **(3)**

(*c*) On the 1st April 1990 a serious fire required an extra $\frac{1}{4}$ million gallons of water from the reservoir to bring the fire under control.
Assuming that the previous trend continues from the new lower level, when will the reservoir run dry if water rationing is not imposed? **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(*a*)	4	1.2		4					1.2.3		Source
(*b*)	3	1.2		3					1.2.4		**1990 Paper 2**
(*c*)	3	1.2		3					1.2.9		**Qu. 10**

(*a*)
- •1 correct scales
- •2 zeros
- •3 graph of $y = \sin\frac{\pi t}{6}$
- •4 graph of $y = -\sin\frac{\pi t}{6}$

(*b*)
- •5 indication of translation $\begin{pmatrix} 0 \\ 1.1 \end{pmatrix}$ to $y = -\sin\frac{\pi t}{6}$
- •6 for minima at $W = 0.1$
- •7 sketch

(*c*)
- •8 indicate on graph effect of fire at $t = 23$
- •9 $t = 26$ (± 1)
- •10 about July($\pm$1) 1990

A function f is defined by the formula $f(x) = 4x^2(x-3)$ where $x \in \mathbf{R}$.

(*a*) Write down the coordinates of the points where the curve with equation $y = f(x)$ meets the x- and y-axes. **(2)**

(*b*) Find the stationary points of $y = f(x)$ and determine the nature of each. **(6)**

(*c*) Sketch the curve $y = f(x)$. **(2)**

(*d*) Find the area completely enclosed by the curve $y = f(x)$ and the x-axis. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
(*a*)	2	1.2	2						1.2.9		Source
(*b*)	6	1.3	6						1.3.12		**1989 Paper 2**
(*c*)	2	1.3	2						1.3.13		**Qu. 1**
(*d*)	4	2.2	4						2.2.6		

(*a*)
- •1 $(0,0)$
- •2 $(3,0)$

(*b*)
- •3 $f'(x) = 12x^2 - 24x$
- •4 $f'(x) = 0$ **stated explicitly**
- •5 $x = 0,\ x = 2$
- •6

x	0^-	0	0^+	2^-	2	2^+
f'	$+$	0	$-$	$-$	0	$+$

- •7 max at $(0,0)$
- •8 min at $(2,-16)$

(*c*)
- •9 correct shape
- •10 $(0,0),(3,0),(2,-16)$ annotated

(*d*)
- •11 $\int_0^3 \left(4x^3 - 12x^2\right)dx$
- •12 area $= -\int_0^3 \left(4x^3 - 12x^2\right)dx$
- •13 $\left[-x^4 + 4x^3\right]_0^3$
- •14 27

ABCD is a quadrilateral with vertices A(4, –1, 3), B(8, 3, –1), C(0, 4, 4) and D(–4, 0, 8).

(*a*) Find the coordinates of M, the midpoint of AB. **(1)**

(*b*) Find the coordinates of the point T, which divides CM in the ratio 2:1. **(3)**

(*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.1
(*a*)	1	0.1					1		0.1		Source
(*b*)	3	3.1					3		3.1.6		1989 Paper 2
(c)	4	3.1					4		3.1.7, 3.1.6		Qu. 2

(*a*) $\bullet^1$ (6,1,1)

(*b*) $\bullet^2$ e.g. $\overrightarrow{CM} = \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$

$\bullet^3$ $\overrightarrow{CT} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$

$\bullet^4$ $T = (4,2,2)$

(*c*) $\bullet^5$ e.g. $\overrightarrow{BT} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$

$\bullet^6$ $\overrightarrow{TD} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} = 2 \times \overrightarrow{BT}$

$\bullet^7$ TD is parallel to BT, T is common point so B, T, D collinear

$\bullet^8$ BT:TD = 1:2

(*a*) (i) Make a sketch of the graph of $y = x^3$, where $-3 \le x \le 3,\ x \in \mathbf{R}$.

(ii) On the same diagram, draw the graph of $y = 6x + 1$. **(3)**

(*b*) State the number of roots which the equation $x^3 = 6x + 1$ has in the interval $-3 \le x \le 3$. **(1)**

(*c*) Calculate the value of the positive root, correct to 3 significant figures. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	3	0.1	3						0.1		Source
(*b*)	1	0.1	1						0.1		**1989 Paper 2**
(*c*)	4	2.1	1	3					2.1.11		**Qu. 3**

(*a*) •1 suitable choice of scales

•2 sketch of $y = x^3$ from $x = -3$ to $x = 3$

•3 sketch of $y = 6x + 1$ from $x = -3$ to $x = 3$

(*b*) •4 3 roots

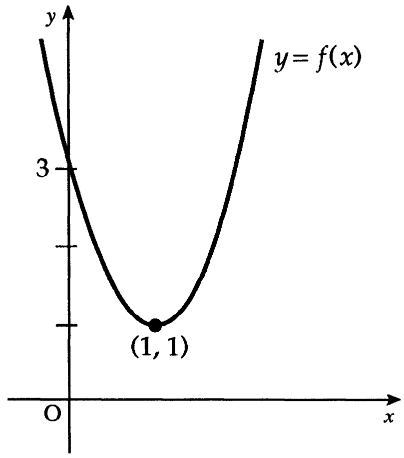
(*c*) •5 1st estimate: between 2 and 3

•6 2nd estimate: between 2.5 and 2.6

•7 3rd estimate: between 2.53 and 2.534

•8 2.53

The diagram shows a sketch of the parabola $y = f(x)$.



(*a*) Copy the sketch of $y = f(x)$. On your diagram, draw the parabola with equation $y = -f(x) + 3$. **(4)**

(*b*) State the values of x for which $3 - f(x) \geq 0$. **(2)**

(*c*) If $g(x) = 3 - f(x)$, express $g(x)$ in terms of x. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(*a*)	4	1.2		4					1.2.4		Source
(*b*)	2	1.2		2					1.2.1		**1989 Paper 2**
(*c*)	3	1.2		3					1.2.7		**Qu. 4**

(*a*)
- •[1] inverted shape
- •[2] passing through origin
- •[3] annotating $(1,2)$
- •[4] annotating $(2,0)$

(*b*)
- •[5] endpoints of $0 \leq x \leq 2$
- •[6] "less than signs" of $0 \leq x \leq 2$

(*c*)
- •[7] $g(x) = ax(x-2)$
- •[8] $(1,2) \Rightarrow 2 = a(1-2)$
- •[9] $g(x) = -2x(x-2)$

An ear-ring is to be made from silver wire and is designed in the shape of two touching circles with two tangents to the outer circle as shown in Diagram 1.

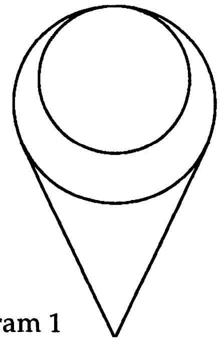


Diagram 1

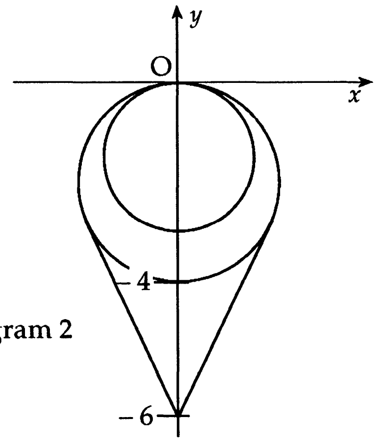


Diagram 2

Diagram 2 shows a drawing of this ear-ring related to the coordinate axes.

The circles touch at (0, 0).

The equation of the inner circle is $x^2 + y^2 + 3y = 0$.

The outer circle intersects the y-axis at (0, –4).

The tangents meet the y-axis at (0, –6).

Find the total length of silver wire required to make this ear-ring. **(6)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		**2.4**
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(-)	6	2.4			6				2.4.2,	2.4.4	**1989 Paper 2 Qu. 5**

(-)

- •[1] radius of inner circle $= \frac{3}{2}$
- •[2] centres are $\left(0, -1\frac{1}{2}\right)$ **and** $(0, -2)$
- •[3] circumferences are 3π **and** 4π
- •[4] *e.g.* $\text{tgt}^2 = 4^2 - 2^2$
- •[5] $\text{tgt} = \sqrt{12}$
- •[6] 29

Some environmentalists are concerned that the presence of chemical nitrates in drinking water presents a threat to health.

The World Health Organisation recommends an upper limit of 50 milligrams per litre (mg/l) for nitrates in drinking water, although it regards levels up to 100 mg/l as safe.

A sub-committee of a Local Water Authority is considering a proposal affecting a small loch which supplies a nearby town with drinking water. The proposal is that a local factory be permitted to make a once-a-week discharge of effluent into the loch, provided that a cleaning treatment of the loch is carried out before each discharge of effluent.

The Water Engineer has presented the following data:

1. The present nitrate level in the loch is 20 mg/l.
2. The cleaning treatment removes 55% of the nitrates from the loch.
3. Each discharge of effluent will result in an addition of 26 mg/l to the nitrate presence in the loch.

and advises the sub-committee that the proposal presents no long-term danger from nitrates to the drinking water supply.

(*a*) Show the calculations you would use to check the engineer's advice. **(5)**

(*b*) Is the engineer's advice acceptable? **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	1.4
(*a*)	5	1.4			5				1.4.1, 1.4.5	Source **1989 Paper 2 Qu. 6**
(*b*)	1	0.1			1				0.1	

(*a*)
- •[1] $u_0 = 20$
- •[2] $u_1 = 35$
- •[3] three further values eg 41.75, 44.78, 46.15
- •[4] 46.76, 47.04, 47.17 looks like approaching a limit
- •[5] five more lead to 47.27'something' $\Rightarrow$ limit = 47.27

(*b*)
- •[6] $47.27 < 50$ so level safe

Diagram 1 is an artist's impression of a new warehouse based on the architect's plans. The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.
With coordinate axes as shown in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.

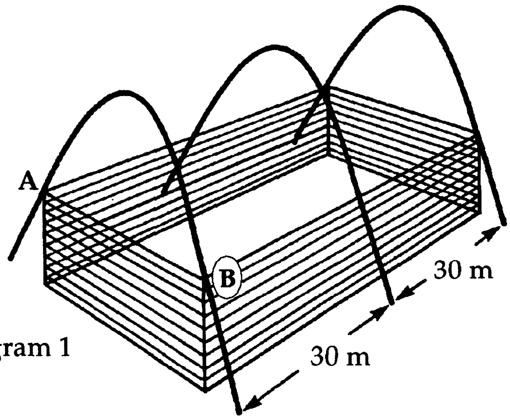


Diagram 1

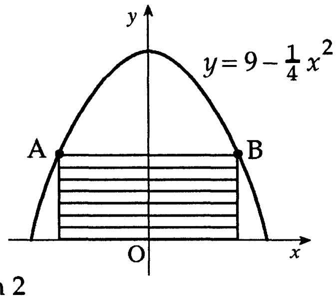


Diagram 2

(*a*) Given that AB is $2x$ metres long, show that the shaded area in Diagram 2 is $\left(18x - \frac{1}{2}x^2\right)$ square metres. **(2)**

(*b*) The architect wished to fit into the girders the cuboidal warehouse which had the maximum volume. Find the value of this maximum volume. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	2	0.1					2		0.1		Source **1989 Paper 2 Qu. 7**
(*b*)	6	1.3					3	3	1.3.15		

(*a*)
- •1 $B = (x, y)$ where $y = 9 - \frac{1}{4}x^2$
- •2 area $= 2x\left(9 - \frac{1}{4}x^2\right)$

(*b*)
- •3 $V = 1080x - 30x^3$
- •4 $\frac{dV}{dx} = 1080 - 90x^2$
- •5 $\frac{dV}{dx} = 0$ **stated explicitly**
- •6 $x = 2\sqrt{3}$
- •7

x	$2\sqrt{3}^-$	$2\sqrt{3}$	$2\sqrt{3}^+$
$\frac{dV}{dx}$	+	0	−

- •8 max at $x = 2\sqrt{3}$ of $1440\sqrt{3}$

A function f is **EVEN** if $f(-x)=f(x)$

e.g. when $f(x)=x^2$, f is **EVEN** because $f(-x)=(-x)^2=x^2=f(x)$.

A function f is **ODD** if $f(-x)=-f(x)$

e.g. when $f(x)=x^3$, f is **ODD** because $f(-x)=(-x)^3=-x^3=-f(x)$.

(*a*) Given that $g(x)=\cos x$ and $h(x)=\sin 2x$, decide for each of the functions g and h whether it is **EVEN** or **ODD**.
Justify your decisions. **(4)**

(*b*) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$ and $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x \, dx$. **(5)**

(*c*) On separate diagrams, draw rough sketches of the graphs of $y=\cos x$ and $y=\sin 2x$ for $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$. **(2)**

(*d*) If $v(x)=x\cos x$, check whether the function v is **EVEN** or **ODD** and

suggest a value for $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x \, dx$. **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	0.1
(*a*)	4	0.1	4						0.1		Source
(*b*)	5	3.2	2	3					3.2.1,	3.2.4	**1989 Paper 2**
(*c*)	2	1.2	2						1.2.3		**Qu. 8**
(*d*)	2	0.1		2					0.1		

(*a*)
- •1 $\cos(-x)=\cos x$
- •2 g is EVEN
- •3 $\sin(-2x)=-\sin(2x)$
- •4 h is ODD

(*b*)
- •5 $\sin x$
- •6 $[\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=2$
- •7 $-\cos 2x$
- •8 $\times\frac{1}{2}$
- •9 $\left[-\frac{1}{2}\cos 2x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=0$

(*c*)
- •10 sketch of $g(x)=\cos x$
- •11 sketch of $h(x)=\sin 2x$

(*d*)
- •12 $v(x)$ is ODD
- •13 0

The formula $d = 200 + 80(\cos 30t° + \sqrt{3}\sin 30t°)$ gives an approximation to the depth of water, d, measured in centimetres, in a harbour t hours after midnight.

(*a*) Express $f(t) = \cos 30t° + \sqrt{3}\sin 30t°$ in the form $k\cos(30t - \alpha)°$ and state the values of k and α, where $0 \le \alpha \le 360$. **(4)**

(*b*) (i) Use your result from part (*a*) to help you sketch the graph of $f(t)$ for $0 \le t \le 12$.

(ii) Hence, on a separate diagram, sketch the graph of d for $0 \le d \le 12$. **(6)**

(*c*) What is the "low-water" time at the harbour during the time interval shown on your graph? **(1)**

(*d*) If the local fishing fleet needs at least 1·5 metres depth of water to enter the harbour without risk of running aground, between what hours must it avoid entering the harbour during the time interval shown on your graph? **(2)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	3.4
(*a*)	4	3.4			4				3.4.1		Source
(*b*)	6	1.2			2	4			1.2.3,	1.2.4	**1989 Paper 2**
(*c*)	1	0.1				1			0.1		**Qu. 9**
(*d*)	2	0.1				2			0.1		

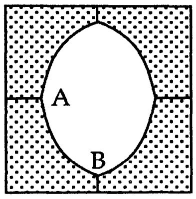
(*a*)
- •1 $k\cos 30t°\cos\alpha° + k\sin 30t°\sin\alpha°$
- •2 $k\cos\alpha° = 1$ **and** $k\sin\alpha° = \sqrt{3}$
- •3 $k = 2$
- •4 $\alpha = 60$

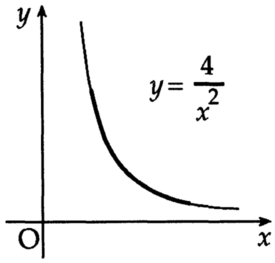
(*b*)
- •5 maximum at (2,2)
- •6 minimum at (8,–2)
- •7 endpoints: (0,1) **or** (12,1)
- •8 graph correctly annotated with 3 points
- •9 sketch with original amplitude increased by factor of 60
- •10 sketch with original graph translated $\begin{pmatrix} 0 \\ 200 \end{pmatrix}$

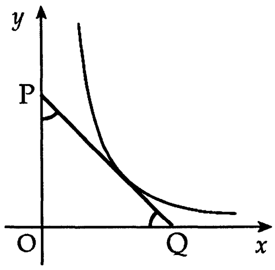
(*c*)
- •11 0800 hours

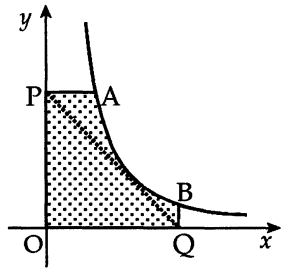
(*d*)
- •12 5.6 hours and 10.4 hours
- •13 e.g. between 5am and 11am

The makers of "OLO", the square mint with the not-so-round hole, commissioned an advertising agency to prepare an illustration to the specification described in (i) to (iii) below. The finished illustration will look like the diagram on the right.









(i) The curve AB in the finished illustration is part of the curve with equation $y = \frac{4}{x^2}$.

(ii) A tangent to this curve, making equal angles with both axes, is to be drawn as shown (line PQ)

(iii) Straight lines perpendicular to the axes are to be drawn from P and Q as shown. The shaded part forms $\frac{1}{4}$ of the finished illustration.

(*a*) State the gradient of PQ and hence find the point of contact of the tangent PQ with the curve. **(5)**

(*b*) Find the equation of PQ and the coordinates of A and B. **(4)**

(*c*) Calculate the shaded area of the finished illustration. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	5	1.1	3	2					1.1.3,	1.1.10	Source
(*b*)	4	1.1	1	3					1.1.7,	0.1	**1989 Paper 2**
(*c*)	6	2.2		6					2.2.6		**Qu. 10**

(*a*)
- •1 $m_{PQ} = -1$
- •2 $f(x) = 4x^{-2}$
- •3 $f'(x) = -8x^{-3}$
- •4 $-8x^{-3} = -1$
- •5 $x = 2$ and $f(2) = 1$

(*b*)
- •6 $x + y = 3$
- •7 $\frac{4}{x^2} = 3$
- •8 $x \approx 1.15$
- •9 A(1.15,3), B(3,0.44)

(*c*)
- •10 suitable division of area
- •11 rectangle $OPA'C' = 3 \times 1.15 = 3.45$
- •12 curved area $QBA'C' = \int_{1.15}^{3} \frac{4}{x^2}\,dx$
- •13 $\left[-\frac{4}{x}\right]_{1.15}^{3}$
- •14 2.15
- •15 $(3.45 + 2.15) \times 4 = 22.4$