Higher Polynomials & Quadratics Exam Revision

1. The remainder when $2x^3 + x^2 - 1$ is divided by $x - 2$ is

A 9 **B** 5 **C** 19 **D** -13

2. The quadratic equation $4kx^2 - 8x + k = 0$ has equal roots.
The value of k, where $k > 0$ is

A 4 **B** 2 **C** 0 **D** -2

3. The remainder when $x^3 - 11x + 10$ is divided by $(x + 3)$ is

A 52 **B** 16 **C** 4 **D** -24

4. The equation $2x^2 + 8 = kx$ has **no real roots**. k must take the values

A ± 8 **B** $-8 < k < 8$ **C** undefined **D** $k < -8$ or $k > 8$

5. For which value(s) of x is the function $f(x) = \dfrac{3}{(x+3)(x-2)}$ undefined?

A 3 **B** 3 and -2 **C** -3 and 2 **D** -6

6. The maximum value of $\dfrac{12}{x^2 - 4x + 10}$ is

A 2 **B** -2 **C** 6 **D** -6

7. A ball is thrown upwards reaching a height of 'h' metres after 't' seconds where $h(t) = 2 + 12t - 3t^2$. The time taken, in seconds, to reach its maximum height is

A 2 **B** 3 **C** 4 **D** 5

8. Given that $x - 1$ is a factor of $x^3 - 6x^2 + px - 6$ then p equals

A -13 **B** -1 **C** 1 **D** 11

9. When $4x^3 + px^2 - x + 1$ is divided by $2x + 1$, the remainder is -1. p is equal to

A -8 **B** -4 **C** -1 **D** 4

10. The set of factors of $2x^3 + 3x^2 - 5x - 6$ contains which of the following factors

(1) $(x+1)$ (2) $(x+2)$ (3) $(2x-3)$

A (1) only **B** (2) only **C** (3) only **D** (1), (2) and (3)

11. If $2x^2 - 12x + 11$ is expressed in the form $2(x - b)^2 + c$, what is the value of c?

A –25 **B** –7 **C** 11 **D** 23

12. The quadratic equation with roots $7 + \sqrt{5}$ and $7 - \sqrt{5}$ can be written as

A $x^2 + 7x + 5 = 0$ **B** $x^2 - 14x + 44 = 0$

C $x^2 - 14x + 24 = 0$ **D** $x^2 + 14x + 44 = 0$

13. Given that $x = -2$ and $x = 1$ are two roots of the equation $x^3 + px^2 - 6x + q = 0$, establish the values of p and q and hence find the third root of the equation. **5**

14. (a) If $k = \dfrac{(x-1)^2}{x^2+4}$, where k is a real number, show clearly that

$$(k-1)x^2 + 2x + (4k-1) = 0.$$ **3**

(b) Hence find the value of k given that the equation $(k-1)x^2 + 2x + (4k-1) = 0$ has equal roots and $k > 0$. **5**

15. Two functions, defined on suitable domains, are given as

$$f(x) = x(x^2 - 1) \quad \text{and} \quad g(x) = x - 1.$$

(a) Show that the composite function, $h(x) = f(g(x))$, can be written in the form $h(x) = ax^3 + bx^2 + cx$, where a, b and c are constants, and state the value(s) of a, b and c. **4**

(b) Hence solve the equation $h(x) = 6$, for x, showing clearly that there is only one solution. **4**

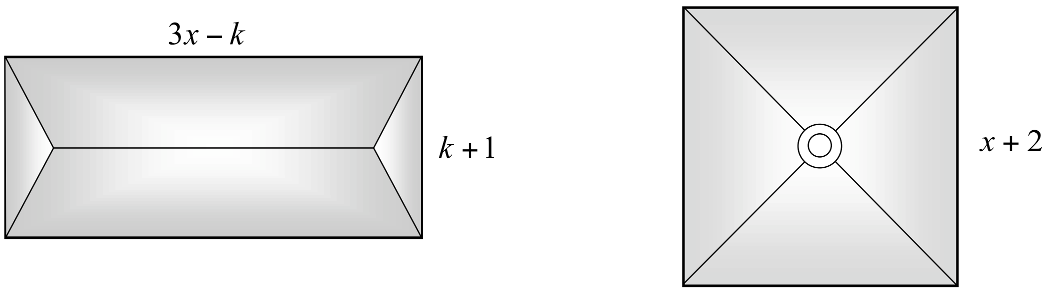
16. A curve has as its equation $y = (p+1)x^3 - 3px^2 + 4x + 1$, where p **is a positive integer**.

(a) Find $\dfrac{dy}{dx}$. **2**

(b) Hence establish the value of p given that this curve has only **one stationary point**. **5**

17. A householder is considering two different designs for a conservatory.

One design has a rectangular base measuring $3x - k$ by $k + 1$ metres and the other design is square based with side $x + 2$ metres. Both x and k are constants.



(a) With both designs having the same base **area**, show clearly that the following equation can be formed.

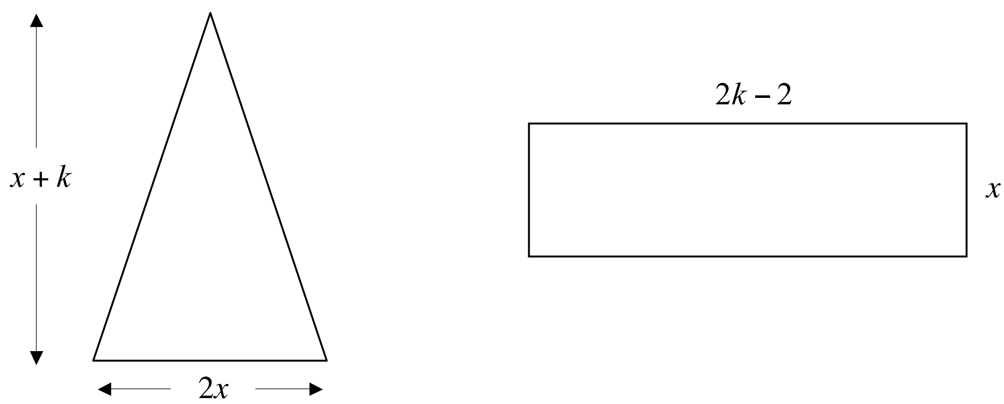
$$x^2 + (1 - 3k)x + (k^2 + k + 4) = 0$$ **3**

(b) Given that the above equation has **equal roots**, find first the value of k, and then the base area of each conservatory in square metres. **5**

18. Express the function $f(x) = 3x^2 - 6x + 11$ in the form $p(x - q)^2 + r$. **3**

19. Consider the isosceles triangle and the rectangle below.

The triangle has a base measuring $2x$ and a vertical height of $x + k$.
The rectangle has dimensions $2k - 2$ by x as shown.
All dimensions are in centimetres.



(a) Given that the **area of the rectangle** is 4cm^2 **more than** the area of the triangle, **show clearly** that the following equation can be formed.

$$x^2 + (2 - k)x + 4 = 0$$ **3**

(b) Hence find k, given that the equation $x^2 + (2 - k)x + 4 = 0$ has equal roots and $k > 0$. **3**

(c) Find x when k takes this value and calculate the area of each shape. **3**

20. Two functions f and h are defined on suitable domains as follows :

$$f(x) = 2x - 2 \quad \text{and} \quad h(x) = \frac{4\frac{1}{2}}{x+2} .$$

(a) Given that $g(x) = f\big(h(x)\big)$ show that $g(x)$ can be written as

$$g(x) = \frac{5 - 2x}{x + 2} .$$ **3**

(b) Hence solve algebraically the equation $g(x) = x^2$. **3**

21. **(a)** If $3x^3 - kx^2 - 38x - 24$ is exactly divisible by $(x + 3)$, find the value of k. **3**

(b) Hence, write the expression in fully factorised form when k takes this value. **2**

22. The equation $kx^2 + (k - 3)x + k = 0$ has equal roots.

Find the value of k given that $k > 0$. **4**

23. An equation is given as $\dfrac{5(k-2)}{x} = x + 2(2 - k)$, where $x \neq 0$.

(a) Show clearly that this equation can be written in the form

$$x^2 + (4 - 2k)x + (10 - 5k) = 0 .$$ **2**

(b) Hence find the values of k which would result in the above equation having **equal roots**. **4**

24. If $x^3 + px + 30$ is exactly divisible by $x - 2$ find the value of p and hence factorise the expression completely. **4**

25. **(a)** If $3x^3 - kx^2 - 38x - 24$ is exactly divisible by $(x + 3)$, find the value of k. **3**

(b) Hence, write the expression in fully factorised form when k takes this value. **2**

26. The equation $kx^2 + (k - 3)x + k = 0$ has equal roots.

Find the value of k given that $k > 0$. **4**