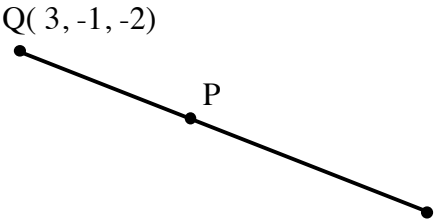
Higher Vectors Exam Revision

1. (*a*) E, F and G have coordinates (1, 4, -2) , (-1, 8, -1) and (-5, 16, 1) respectively.

(i) Write down the components of $\overrightarrow{EF}$. **1**

(ii) Hence show that the points E, F and G are collinear. **3**

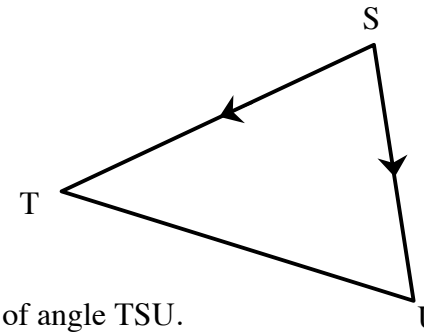
(*b*) The point P divides QR in the ratio 2:3 as shown in the diagram.



Find the coordinates of P. **3**

2. The diagram shows triangle STU where

$\overrightarrow{ST} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\overrightarrow{SU} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$



(*a*) Find the value of $\overrightarrow{ST} \cdot \overrightarrow{SU}$. **1**

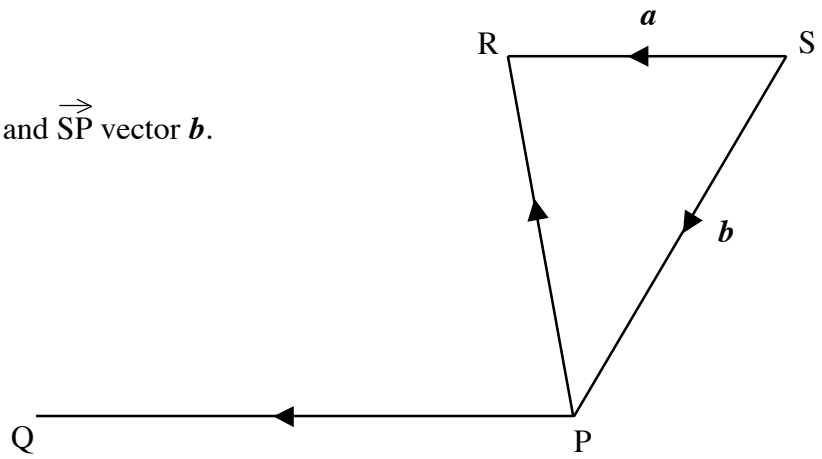
(b) Use the result of (*a*) to find the size of angle TSU. **4**

3. Two vectors are defined as $V_1 = \sqrt{6}\,\boldsymbol{i} + \boldsymbol{j} + \sqrt{8}\,\boldsymbol{k}$ and $V_2 = 4\,\boldsymbol{i} + \sqrt{24}\,\boldsymbol{j} + a\sqrt{3}\,\boldsymbol{k}$, where a is a constant and all coefficients of $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are greater than zero.

Given that the vectors V_1 and V_2 are perpendicular, calculate the value of a. **4**

4. Consider the vector diagram.

$\overrightarrow{SR}$ represents vector $\boldsymbol{a}$ and $\overrightarrow{SP}$ vector $\boldsymbol{b}$.
Angle PSR = $60°$.



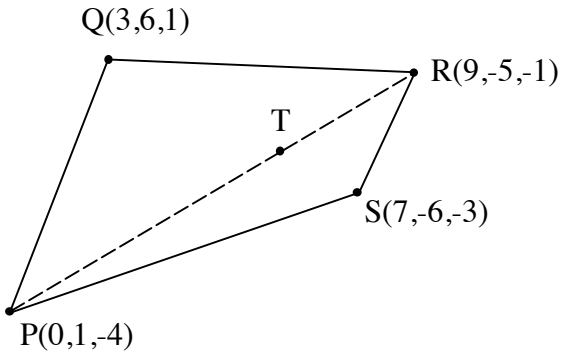
(a) Express displacement $\overrightarrow{PR}$ in terms of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. **1**

(b) Given that vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ have magnitudes of 2 units and 3 units respectively, evaluate the scalar product $\boldsymbol{a} \cdot \boldsymbol{b}$. **2**

(c) Hence evaluate the scalar product $\boldsymbol{v} \cdot \boldsymbol{u}$ when $\boldsymbol{v} = \overrightarrow{PQ} = 2\boldsymbol{a}$ and $\boldsymbol{u} = \overrightarrow{PR}$. **5**

5. Quadrilateral PQRS has vertices P(0,1,-4) , Q(3,6,1) , R(9,-5,-1) and S(7,-6,-3).

T is a point on diagonal PR.



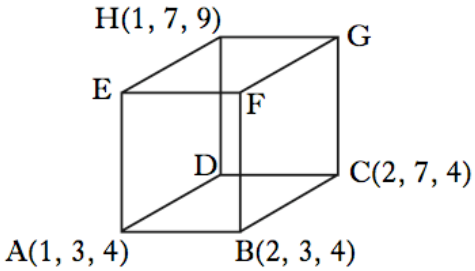
(a) Given that $\frac{PT}{TR} = \frac{2}{1}$, establish the coordinates of T. **4**

(b) Prove that Q, T ans S are collinear and state the ratio in which T divides QS. **4**

6. L, M and N are the points (2, 0, -1), (4, -6, 8) and (-5, 7, 0) respectively.

Calculate the size of angle LMN. **5**

7. The diagram shows a wire framework in the shape of a cuboid with the edges parallel to the axes.



Relative to these axes, A, B, C and H have coordinates (1, 3, 4), (2, 3, 4), (2, 7, 4) and (1, 7, 9) respectively.

(a) State the lengths of AB, AD and AE. **1**

(b) Write down the components of HB and HC and hence or otherwise calculate the size of angle BHC. **7**

8. Given that $\boldsymbol{a} \cdot (\boldsymbol{a} + \boldsymbol{b}) = 26$, $|\boldsymbol{a}| = 4$ and $|\boldsymbol{b}| = 5$, calculate the size of the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. **3**

Question 1

(a) i) $\vec{EF} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$

ii) FG = 2EF

$\therefore$ FG and EF are parallel
Common point F
$\therefore$ collinear

(b) P(-3 ,3 ,-4)

Question 2

(a) $\vec{ST}.\vec{SU} = 2$ (b) $79 \cdot 1^{o}$

Question 3

- 1 $V_1 = \begin{pmatrix} \sqrt{6} \\ 1 \\ \sqrt{8} \end{pmatrix}$, $V_2 = \begin{pmatrix} 4 \\ \sqrt{24} \\ a\sqrt{3} \end{pmatrix}$
- 2 If perp. than $V_1 . V_2 = 0$
- 3 $V_1 . V_2 = 4\sqrt{6} + \sqrt{24} + a\sqrt{24} = 0$
- 4 $4\sqrt{6} + 2\sqrt{6} + 2a\sqrt{6} = 0$
  $\therefore \ 2a = -6 \ \rightarrow \ a = -3$

Question 4

(a)
- 1 $\vec{PR} = a - b$

(b)
- 1 $a.b = |a||b| \cos\theta$ (or equiv)
- 2 $a.b = 2 \times 3 \times \frac{1}{2} = 3$

(c)
- 1 $v.u = 2a.(a - b)$
- 2 $v.u = 2a.a - 2a.b$
- 3 $v.u = 2|a|^2 - 2a.b$
- 4 $v.u = 2(2^2) - 2(3)$
- 5 $v.u = 2$

Question 5

(a)
- 1 $PT = 2TR$
- 2 $\underset{\sim}{t} - \underset{\sim}{p} = 2(\underset{\sim}{r} - \underset{\sim}{t})$
- 3 $3t = 2r + p$

• 4 T(6,-3,-2)

(b) • 1 $\vec{QT}$ *and* $\vec{TS}$

• 2 $\vec{QT} = \begin{pmatrix} 3 \\ -9 \\ -3 \end{pmatrix}$, $\vec{TS} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$

• 3 Since $QT=3TS$, and T is a common point, then Q,T & S are collinear.

• 4 in the ratio 3 : 1

Question 6 ans: 30·5°

_1	finds $\overrightarrow{ML}$ and $\overrightarrow{MN}$	_1	$\overrightarrow{ML} = \begin{pmatrix} -2 \\ 6 \\ -9 \end{pmatrix}$; $\overrightarrow{MN} = \begin{pmatrix} -9 \\ 13 \\ -8 \end{pmatrix}$
_2	finds magnitudes	_2	$\lvert\overrightarrow{ML}\rvert = 11$; $\lvert\overrightarrow{MN}\rvert = \sqrt{314}$
_3	finds scalar product	_3	$\overrightarrow{ML}.\overrightarrow{MN} = 18 + 78 + 72 = 168$
_4	substitutes into formula	_4	$\cos\theta = \frac{168}{11\sqrt{314}}$
_5	finds angle	_5	$\theta = 30\cdot5°$

Question 7

•1 $AB = 1, AD = 4, AE = 5$

•2 $\overrightarrow{HB} = \begin{pmatrix} 1 \\ -4 \\ -5 \end{pmatrix}$

•3 $\overrightarrow{HC} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$

•4 $\overrightarrow{HB}.\overrightarrow{HC} = 1 + 0 + 25 = 26$

•5 *magnitude of* $HB = \sqrt{42}$

•6 *magnitude of* $HC = \sqrt{26}$

•7 $\cos HBC = (26) / \sqrt{42}\sqrt{26}$

•8 $H\hat{B}C = 38\cdot1\,°$ *or* $0\cdot665$ radians

Questions 10

ans: 60°

_1	multiplies out brackets	_1	$\boldsymbol{a}\,.\,\boldsymbol{a} + \boldsymbol{a}\,.\,\boldsymbol{b}$
_2	substitutes values	_2	$4^2 + 4\times5\times\cos\theta = 26$
_3	finds values for $\cos\theta$ and finds θ	_3	$\cos\theta = \frac{1}{2}; \theta = 60°$